Local Public Goods: First Best Allocations and Supporting Prices*

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Abstract

For a model with local public goods we prove that, for any first best allocation, there exists a system of personalized prices and lump sum transfers between consumers that will support that first best allocation as an equilibrium. Consumers are free to migrate between regions. The form of personalised prices and lump sum transfers required depends on how each consumer's preferences change as any consumer migrates. It is demonstrated that consumers not only must face a system of personalized prices in equilibrium but must also, in general, face a different system of personalized prices out of equilibrium. Selections from the set of Pareto optimal allocations are made to prove the existence of an equilibrium.

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1 Introduction

For a model with local public goods we prove that, for any first best allocation, there exists a system of personalized prices and lump sum transfers between consumers that will support that first best allocation as an equilibrium. Lindahl (1958) and Samuelson (1954) showed a condition characterising first best allocations of pure public goods is that the sum of the marginal rates of substitution of all consumers, between each public good and each private good, is equal to the corresponding marginal rate of transformation. The relative price of any public good and any private good that any consumer faces needed to support any first best allocation is just the marginal rate of substitution between that public good and private good. In this sense the prices of pure public goods must be personalised. We extend the Lindahl-Samuelson prices to ensure that the demand for each local public good in each region is equal to supply. It is demonstrated that consumers not only must face a system of personalized prices in equilibrium but must also, in general, face a different system of personalized prices out of equilibrium.[1]

Tiebout (1956) theorised that with a sufficiently large number of regions migration would lead to near efficient provision of local public goods. We consider a class of models that may have a small number of regions. Where all consumers are charged the same price for each public good, Tiebout claimed that migration would ensure that consumers with the same demand will reside in the same region. If the number of regions is too small, or production opportunities or consumer satisfaction depend on the number or identity of consumers in each region then, with uniform public goods prices, migration need not ensure that consumers of the same demand reside together.

Since Tiebout's seminal paper a number of papers have sought to clarify when Tiebout equilibria exist and when Tiebout equilibria are Pareto optimal. Ellickson (1979) models each region as charging a different regional price, but the same price of each consumer in each region. Like Ellickson, Wooders (1978, 1980) modelled each region as financing its expenditure on one local public good by charging each consumer the same regional price for that public good. Bewley (1981) assumes that regional governments finance

[1] These prices have more recently been called "conjectural" prices. Consumers have to know that the prices they face may be different out of equilibrium so that the equilibrium allocation is the "best" affordable allocation under the equilibrium allocation of consumers amongst regions and the "out of equilibrium" allocation of consumers amongst regions.

public goods (services) through uniform regional wealth taxes. In general the revenue mechanisms outlined in Bewley, Ellickson and Wooders will not lead to the implementation of first best allocations.

The only existence proofs for models of local public good economies with personalised prices, to the author's knowledge, are those of Greenberg (1983) and Wooders (1981, 1989). Greenberg (1983) shows the existence of equilibrium in a local public goods economy without migration for a continuum of consumers. (In fact Greenberg (1983) does allow for a restrictive form of migration. Any consumer is free to migrate to another region if all consumers in that region will accept him.)

Wooders (1989) proves existence of equilibrium for a Tiebout economy with Lindahl and personalised lump sum taxes (or subsidies). This is done by replicating an economy with a finite number of consumers and showing allocations in the core of a sufficiently replicated economy to be "equilibrium" allocations.

Little research effort has been directed to when the First or Second Welfare Theorem hold or when equilibria exist in finite local public good models. The existence proofs for continuum or replicated models suggest, but do not imply, the existence of equilibrium for finite local public good models.

In Section 2 I introduce a class of local public good models. Two candidate price spaces are considered. Each price space is the analogue of a price space in Wooders (1989). In the first price space each consumer is charged the same personalised price for each public good, whatever his (her) choice of residence or the residence choice of other consumers. Non-anonymous crowding in consumption and production is allowed. Crowding is non-anonymous if consumer identity may affect utility and/or production opportunities. The residence of each consumer in any region is priced using personalised lump sum taxes (subsidies). Each consumers lump sum tax (subsidy) is independant of the residence choice of any other consumer. The second price space is characterised by complete personalised prices. Each consumer is charged a different personalised price for each public good as his (her) choice of residence changes or the residence choice of other consumers changes. Consumers may also be charged a personalised lump sum tax (subsidy) that changes as his (her) choice of residence changes or the residence choice of other consumers changes.[2] Commodities are indexed by the location (partition) of all

[2]Both the complete personalized prices and lump sum transfers are "conjectural".

consumers among regions.

Section 3 gives an example of a model that illustrates the Theorems to follow. The example rules out personalised prices as a candidate for the implementation of all first best allocations in local public good models when the same allocations of private and local public goods under some other assignment of consumers to regions yields some consumer higher utility. The example illustrates how a weakly efficient allocation may be implemented by a complete personalised price system. As a consequence the equivalence between Lindahl prices and personalised lump sum taxes (subsidies) and complete personalised prices without lump sum taxes (subsidies), proved in Wooders (1989) for sufficiently replicated economies, is shown not to hold for finite models.

In Section 4 three Second Welfare theorems are offered. In the first Second Welfare Theorem non-anonymous crowding in consumption and production may occur. In the second Second Welfare Theorem the crowding in consumption is restricted. Consumers may not enjoy higher utility associated with the same allocations as the assignment of consumers to regions changes. In addition, production opportunities do not expand as consumers move away from the Pareto optimal partition. As a consequence it is shown that the price space may be constrained to the set of personalised prices without lump sum taxes (subsidies). In the third Second Welfare Theorem consumer preferences are further restricted in that they must be locally the same. The price space may be constrained to the set of non-personalised prices without lump sum taxes (subsidies).

Section 5 presents existence results and Section 6 demonstrates that if any equilibrium allocation is an efficient allocation in a "global" sense, equilibrium at a system of complete prices is in the core.

All proofs are in the appendices.

2 The Model

We consider an economy with M private goods, G public goods, I consumers and J regions. We use the convention $\mathcal{M} = \{1, \ldots, m, \ldots, M\}$, and similarly for $\mathcal{G}$, $\mathcal{I}$ and $\mathcal{J}$.

2.1 Consumers

Consumer residence choice is indicated by a partition of the set of all consumers $\mathcal{I}$, S, where $\#S = J$. The set of all partitions of $\mathcal{I}$, such that $\#S = J$, is denoted by Z.

The consumption of local public goods by all consumers residing in region j is $g^j \in \Re^G$. Sometimes it is convenient to write the consumption of local public goods by all consumers residing in region j as $\underline{g}^j = (0, \ldots, g^j, \ldots, 0) \in \Re^{GJ}$. The consumption vector of consumer i when residing in region j is $x^i_S = (\underline{g}^j_S, l^i_S)$, where l^i_S is the consumption vector of private goods and $\underline{g}^j_S$ is the consumption vector of public goods relative to the partition S. Aggregate consumption is $x_S = (g_S, l_S)$, where $g_S = (g^j_S)$ is the consumption of local public goods by all consumers and $l_S = \sum_i l^i_S$ is the aggregate consumption of private goods.

Consumers are initially endowed with private goods and a residence. The endowment of consumer i is w^i. The partition under which each consumer initially resides is S_w. The aggregate endowment is $w = \sum_i w^i$.

Each consumer i has a consumption set over the space of local public goods and private goods relative to the partition S, $X^i_S \subset \Re^{GJ+M}$. The consumption set over the space of local public goods, private goods and the partitions of consumers is $X^i \subset \Re^{GJ+M} \times Z$. The preferences of consumer i are represented by a complete preordering $\succ^i$ over X^i.

Given the preferences of consumer i the *better than*, *worse than* and *strictly better than* sets relative to the partition of consumers S are defined as follows:

$$R^i_S(x_{S'}) = \{z_S \in X^i_S | z_S \succeq^i x_{S'}\},\ L^i_S(x_{S'}) = \{z_S \in X^i_S | x_{S'} \succeq^i z_S\}$$
$$\text{and } P^i_S(x_{S'}) = \{z_S \in X^i_S | z_S \succ^i x_{S'}\}.$$

2.2 Production

Production is constant returns to scale. Prices are chosen such that there are no profits. The set of production opportunities in the space of local public goods and private goods relative to the partition S is denoted by $Y_S \subset \Re^{GJ+M}$. The marginal rate of transformation between private goods is independant of partition. The production set over the space of local public goods, private goods and the partitions of consumers is $Y \subset \Re^{GJ+M} \times Z$. The net output of local public goods and private goods relative to the partition S is y_S.

An allocation $((x^{\star i}_{S^\star}), y^\star_{S^\star})$ is *feasible* if

(1) $y^\star_{S^\star}$ is in $Y_{S^\star}$ and for all i, $x^{\star i}_{S^\star}$ is in $X^i_{S^\star}$,
(2) $y^\star_{S^\star} + w = x^\star_{S^\star}$.

2.3 Weak Efficiency

An allocation $((x^{\star i}_{S^\star}), y^\star_{S^\star})$ is *weakly efficient*[3] if conditions (1), (2) and (3) hold where condition (3) is as follows:

(3) if there exists an allocation $((x'^i_{S'}), y'_{S'})$ such that $x'^i_{S'} \succ^i x^{\star i}_{S^\star}$ for all i then $((x'^i_{S'}), y'_{S'})$ is not feasible.

The *weakly preferred set* defined with respect to the consumption of all consumers $(x'^i_{S'})$, relative to the partition S, $P_S((x'^i_{S'}))$ is the sum of the strictly preferred set of each consumer, $P_S((x'^i_{S'})) = \sum_i P^i_S(x'^i_{S'})$.

2.4 Prices

The price space, $\triangle$, consists of a price for every local public good relative to the partition of consumers, p_S, a price for every private good, q, and a lump sum transfer, τ_S. Let the price space $\triangle = \{(p, q, \tau) \in \Re^{GJ+M}_+ \times \Re | (p, q, \tau) \neq 0\}$. Denote the price of every local public good and private good, (p_S, q), by v_S and let the price of every local public good $p_S = (p^{gj}_S)$ where p^{gj}_S is the price paid for public good g in region j under the partition S. Let $q = (q^m)$ where q^m is the price paid for private good m.

2.4.1 Non-Personalised, Personalised and Complete Personalised Prices

When prices are *complete personalised* all local public good prices are personalized and free to adjust as consumers migrate. If prices are complete

[3] Diamantaras and Wilkie (1992, Theorem 1, p. 4) prove that, in general, the set of weakly efficient allocations is larger than the set of Pareto optimal allocations, defined with reference to the notion of Pareto optimality used by Debreu (1959, p. 91) and Milleron (1972, p. 427). The notion of weakly efficient allocations used here is equivalent to the notion of Pareto optimality used in Foley (1970, p. 67).

personalised then the price vector that consumer i faces under the partition S is v^i_S and the lump sum transfer that consumer i pays under the partition S is τ^i_S, where (v^i_S, τ^i_S) is an element of $\triangle$.

When prices are *personalised* consumers are charged different prices for the same local public good but these will not adjust as consumers migrate. In addition, where ever consumer i resides consumer i pays (or is payed) a lump sum τ^i. However, the prices that consumers face do not change as the partition of consumers changes. If prices are personalised then the price vector that consumer i faces is v^i where (v^i, τ^i) is an element of $\triangle$.

When prices are *non-personalised* there are no lump sum payments and each consumer pays the same price for any public good. In addition the prices that consumers face do not change as partitions change. If prices are non-personalised then the price vector that consumer i faces is v where $(v, 0)$ is an element of $\triangle$.

Let $u = (\sum_i p^i, q)$.

Remark: The notion of complete personalised prices has re-appeared in work subsequent to this work; in Gilles and Scotchmer (1995, 1997) and Manning (1994, 1995).

In Gilles and Scotchmer (1995, 1997) the set of agents is denoted by a Lebesgue measurable set $A \subset [0,1]$. In the notation of Gilles and Scotchmer, let Σ be the σ-algebra of all Lebesgue measurable subsets of A and let $\mu : \Sigma \to [0,1]$ be a regular Borel probability measure on $(A, \sum)$. Let Γ be a non-empty algebra of potential clubs. A Γ-*coalition structure* is defined as a finite Γ-partition of A a.e. (i.e. a finite collection $\mathcal{P}$, where each $E \in \mathcal{P}$ is a coalition in Γ, is a Γ-coalition structure on A if $\mu(E \cap F) = 0$ for all E, $F \in \mathcal{P}$, if $\mu(E) > 0$ for each $E \in \mathcal{P}$, and if $\sum_{E \in \mathcal{P}} \mu(E) = 1$).

A *club* is a pair (E, y) consisting of a *coalition* $E \in \Gamma$ and *public facilities* $y \in \mathcal{Y}$. A *club structure* is a collection $K \subset \{(E, y) \mid E \in \Gamma \text{ and } y \in \mathcal{Y}\}$.

Private goods prices are in the space $\Delta := \{p \in \Re^l_+ | \sum\limits_{i=1}^{l} p_i = 1\}$, where l is the number of private goods. Prices for private goods are defined as a function $p : K \to \Delta$ and, in addition, there are personalized admission prices $V_a : K \to \Re$, $a \in A$. Taken together the prices for private goods and admission prices are called *conjectural prices.*

Complete personalized prices, in the sense defined in this paper, can be obtained from conjectural prices, in the sense of Gilles and Scotchmer (1995, 1997), by letting A be some finite subset of $[0,1]$, $\Sigma = 2^A$, μ be the normalized counting measure, $\mathcal{Y} = \Re^{GJ}$, $p : K \to \Delta$ be a constant function and $V_a : K \to \Re$ be linear in $\Re^{GJ}$, for all $a \in A$, such that $V_a(K) = \sum_{g \in G, j \in J} p_E^{agj} \underline{g}_E^j + \tau_E^a$.

2.5 Equilibrium

An allocation $((\hat{x}^i_{\hat{S}}), \hat{y}_{\hat{S}})$ is a *quasi-equilibrium relative to a complete personalised price system* $(\hat{v}^i_S), (\hat{\tau}^i_S)$ if and only if it is feasible and if

(1) for all S and every y_S in Y_S, $\hat{u}_{\hat{S}} \hat{y}_{\hat{S}} \geq \hat{u}_S y_S$,

(2) for all i, for all S and for every x^i_S in X^i_S such that $x^i_S \succeq^i \hat{x}^i_{\hat{S}}$, $\hat{v}^i_S x^i_S + \hat{\tau}^i_S \geq \hat{v}^i_{\hat{S}} \hat{x}^i_{\hat{S}} + \hat{\tau}^i_{\hat{S}}$.

If (1) and (2′) hold and $((\hat{x}^i_{\hat{S}}), \hat{y}_{\hat{S}})$ is feasible then it is an *equilibrium relative to a complete personalized price system.*

(2′) for all i, for all S and for every x^i_S in X^i_S such that $x^i_S \succ^i \hat{x}^i_{\hat{S}}$, $\hat{v}^i_S x^i_S + \hat{\tau}^i_S > \hat{v}^i_{\hat{S}} \hat{x}^i_{\hat{S}} + \hat{\tau}^i_{\hat{S}}$.

An allocation $((\hat{x}^i), \hat{y})$ is a *quasi-equilibrium relative to a personalised price system* $(\hat{v}^i), (\hat{\tau}^i)$ or a *non-personalised price system* $(\hat{v})$ if and only if it is feasible and (1) and (2) hold. Similarly, an allocation $((\hat{x}^i), \hat{y})$ is an *equilibrium relative to a personalised price system* $(\hat{v}^i), (\hat{\tau}^i)$ or a *non-personalised price system* $(\hat{v})$ if and only if it is feasible and (1) and (2′) hold.

3 Example

In example 3.1 the marginal conditions of Samuelson (1954) have no meaning (here marginal rates of substitution are sometimes infinite), however the general implications are clear. In general there will not exist a personalised price vector with partition invariant Lindahl and lump sum transfers that supports any weakly efficient allocation as an equilibrium relative to a personalised price system. However for any weakly efficient allocation there does exist a complete personalised price vector that supports that allocation as an equilibrium relative to a complete personalised price system.

Example 3.1 There are two consumers indexed by $i \in \mathcal{I} = \{1, 2\}$, two regions indexed by $j \in \mathcal{J} = \{1, 2\}$, one private good (leisure) denoted by $l \in \Re$ and one local public good denoted by g. There are two partitions

$\{\{1,2\},\emptyset\}$ and $\{\{1\},\{2\}\}$ denoted by S_1 and S_2 respectively. Assume each consumer is endowed with one unit of leisure and both consumers are assumed to initially reside in region 1. The preference preordering of consumer 1 is represented by $U^1(g,l) = g$ under the partition S_1 and $U^1(g,l) = 2/3g$ under the partition S_2. The preference ordering of consumer 2 is represented by $U^2(g,l) = g + 3l$ under both partitions. Each consumer has a consumption set $X^i = \Re^2_+ \times [0,1] \times Z$.

The production opportunities under each partition are:

$$Y_{S_1} = \{y_{S_1} \in \Re^3_+ | y_{S_1} + w_{S_1} = (g^1, 0, l^1) \text{ and } g^1 + l^1 \leq 2\},$$

$$Y_{S_2} = \{y_{S_2} \in \Re^3_+ | y_{S_2} + w_{S_2} = (g^1, g^2, l^1 + l^2) \text{ and } g^1 + l^1 \leq 1, g^2 + l^2 \leq 1\},$$

where $Y = \prod_Z Y(S_k)$.[4]

All weakly efficient allocations are associated with joint residence. Consider the weakly efficient allocation denoted by $((x^1_J, x^2_J), y_J)$ or just J, where $y_J = ((3/2, 0), (-3/2))$, $x^1_J = ((3/2, 0), (0))$ and $x^2_J = ((3/2, 0), (1/2))$.

Restrict attention to the set of all non-negative personalised prices with no lump sum transfers denoted by $\triangle^{L,\tau=0}$.

$U^1(x^1_J) = 3/2$ and $U^2(x^2_J) = 3$. Therefore we can identify the weakly preferred set relative to x^1_J and x^2_J :

$$\begin{aligned} P_{S_1}(J) &= \{x_{S_1} \in X_{S_1} | g^1 > 3/2, g^1 + 3l^2 > 3\}, \\ P_{S_2}(J) &= \{x_{S_2} \in X_{S_2} | g^1 > 1, g^2 + 3l^2 > 3\}, \end{aligned}$$

where $P(J) = \prod_Z P_{S_k}(J)$. Normalize the wage rate to be unity.

Personalised Prices

Restrict attention to the set of all non-negative personalised prices with no lump sum transfers. Inspection of Y_{S_1} and $P_{S_1}(J)$ reveals that under the

[4]Suppose that $\{A_{S_1}, \ldots, A_{S_m}\}$ is a collection of sets indexed by the partitions $S_1, \ldots, S_m$. Let $X = A_{S_1} \cup \ldots \cup A_{S_m}$. We define the *star product* of this indexed collection of sets denoted by $\Pi_{S_k} A_{S_k}$, to be the set of all m-tuples $(0, \ldots, (x_{S_k}), \ldots, 0)$ of elements of X such that $x_{S_k} \in A_{S_k}$, for each k.

partition S_1 the supporting price vector that consumers 1 and 2 face are constrained to be elements of $\triangle^{L,\tau=0}_{S_1,1}$, and $\triangle^{L,\tau=0}_{S_1,2}$, where

$$\triangle^{L,\tau=0}_{S_1,1} = \{p \in \Re^4_+ | \, p = ((2/3, 2/3), (1), (0))\},$$
$$\triangle^{L,\tau=0}_{S_1,2} = \{p \in \Re^4_+ | \, p = ((1/3, 1/3), (1), (0))\}.$$

Inspection of Y_{S_2} and $P_{S_2}(J)$ reveals that, under the partition S_2, the supporting price vectors that both consumers must face so that the equilibrium allocation is the "best" affordable allocation is an element of $(\triangle \times \triangle)^{L,\tau=0}_{S_2}$, where

$$(\triangle \times \triangle)^{L,\tau=0}_{S_2} = \{(p^1, p^2) \in \triangle \times \triangle | \, p^1 = ((\alpha, \alpha), (1), (0)),$$
$$\text{and } p^2 = ((\beta, \beta), (1), (0)); \alpha \in [1, \infty), \; \beta \in [1/3, 1] \text{ and } \beta \leq 1 - \alpha\}.$$

Clearly $(\triangle^{L,\tau=0}_{S_1,1} \times \triangle^{L,\tau=0}_{S_1,2}) \cap \, (\triangle \times \triangle)^{L,\tau=0}_{S_2} = \emptyset$ and so J cannot be supported with non-negative personalised prices with no lump sum transfers. Expanding the set of admissible personalised prices to include lump sum transfers does not allow for the implementation of the weakly efficient allocation J. To see this consider the expanded set of admissible prices, denoted by $\triangle^L$, where partition invariant lump sum taxes (subsidies) are allowed.

If consumers reside in different regions then neither consumer provides a service (disservice) to the other, so there are no lump sum transfers payed. Therefore, the supporting price vector under the partition S_2 remains the same.

However, because this is a pure public good model with no crowding in production the marginal rate of transformation between the pure public good and leisure in any region is invariant to the number of residents in that region. Denote the closure of the weakly preferred sets by $Cl\, P_{S_k}(J)$. For the partition S_1, $Y_{S_1} \cap Cl\, P_{S_1}(J) \neq \emptyset$, so the sum of the personalised prices that both consumers must face is determined by the marginal rate of transformation. That is the sum of the personalised prices must be unity. Therefore, under the partition S_1 the set of admissible prices that consumers 1 and 2 face must be

$$\triangle^{L}_{S_1,1} = \{p \in \Re^4_+ | \, p = ((2/3, 2/3), (1), (\tau^1))\},$$
$$\triangle^{L}_{S_1,2} = \{p \in \Re^4_+ | \, p = ((1/3, 1/3), (1), (\tau^2))\}.$$

Clearly, $(\triangle^L_{S_1,1} \times \triangle^L_{S_1,2}) \cap (\triangle \times \triangle)^L_{S_2} = \emptyset$. Therefore, the expansion of the set of admissible personalised prices does not allow for the implementation of J.

Complete Personalised Prices

Any weakly efficient allocation, in this model, may be supported by non-negative complete personalised prices. The complete personalised prices may be constructed from the prices that lead to separation of the production and Pareto preferred sets in the previous section.

Let the set of admissible complete personalised prices without lump sum transfers under each partition $S_k,\ k = 1, 2$ be denoted by $\triangle^{C,\tau=0}_{S_k}$.

Clearly,

$$\triangle^{C,\tau=0}_{S_1,1} = \{p \in \Re^4_+ |\ p = ((2/3, 2/3), (1), (0))\},$$
$$\triangle^{C,\tau=0}_{S_1,2} = \{p \in \Re^4_+ |\ p = ((1/3, 1/3), (1), (0))\},$$

and

$$(\triangle \times \triangle)^{C,\tau=0}_{S_2} = \{(p^1, p^2) \in \triangle \times \triangle |\ p^1 = ((\alpha, \alpha), (1), (0)),$$
$$p^2 = ((\beta, \beta), (1), (0)); \alpha \in [0, \infty), \beta \in [1/3, 1] \text{ and } \beta \leq 1 - \alpha\}.$$

All complete personalised price vectors in $\triangle^{C,\tau=0}_{S_1,1} \times \triangle^{C,\tau=0}_{S_1,2} \times (\triangle \times \triangle)^{C,\tau=0}_{S_2}$ will support J.

4 Second Welfare Theorems

That any weakly efficient allocation $((\hat{x}^i_{\hat{S}}), \hat{y}_{\hat{S}})$ may be implemented as an equilibrium relative to a complete personalised price system is demonstrated for the class of economies that satisfy the following assumptions for each partition S in Z, in Theorem 1:

a.1S for every i, X^i_S is convex and comprehensive,[5]

a.2S for every i and every $\hat{x}^i_{\hat{S}}$ in $X^i_{\hat{S}}$ the sets $L^i_S(\hat{x}^i_{\hat{S}})$ and $R^i_S(\hat{x}^i_{\hat{S}})$ is closed in X^i_S,

a.3S for every i, if x^i_{1S} and x^i_{2S} are two points of X^i_S and if $t \in (0, 1)$, $x^i_{1S} \succ^i x^i_{2S}$ implies $tx^i_{1S} + (1-t)x^i_{2S} \succ^i x^i_{1S}$,

[5] A set A is *comprehensive* if, for any $u \in A$ and any $v \geq u$, $v \in A$.

a.4S for every i and for any x'^i_S in X^i_S such that $x'^i_S > \hat{x}^i_S$, $x'^i_S \succ^i \hat{x}^i_S$,[6]

a.5S Y_S is a convex cone with vertex at the origin,

a.6S for every i, w^i is in X^i_S,

a.7S if y_S is in Y_S and $y_S \neq 0$ at least one $l^m_S < 0$,

a.8S if (g, l) is in Y_S and $g'^{gj}_S = g^{gj}_S$ when $g^{gj}_S \geq 0$ and $g'^{gj}_S = 0$ when $g^{gj}_S < 0$ then $(\underline{g}', l)$ is in Y_S.

Assumptions $(6S)_Z$ allow people to migrate while keeping their endowments.

Theorem 1 *Under* $(1S, \ldots, 8S)_Z$*, if* $((\hat{x}^i_{\hat{S}}),\ \hat{y}_{\hat{S}})$ *is weakly efficient there exists a price vector* $(\hat{v}^i_S)$ *and lump sum transfers* $(\hat{\tau}^i_S)$*, where* $(\hat{v}^i_S, \hat{\tau}^i_S)$ *is in* $\triangle$ *for every* i*,* $\hat{\tau}^i_{\hat{S}} = 0$ *for all* i *and* $\sum_i \hat{\tau}^i_S = 0$ *for all* S*, with non-negative public and private goods prices, such that* $((\hat{x}^i_{\hat{S}}),\ \hat{y}_{\hat{S}},\ (\hat{v}^i_S), (\hat{\tau}^i_S))$ *is a quasi-equilibrium relative to a complete personalised price system.*

If, in addition, $\hat{v}^i_{\hat{S}}\hat{x}^i_{\hat{S}} \neq \inf \hat{v}^i_S X^i_S$*, for all* i *and all* S*, then* $((\hat{x}^i_{\hat{S}}),\ \hat{y}_{\hat{S}},\ (\hat{v}^i_S), (\hat{\tau}^i_S))$ *is an equilibrium relative to a complete personalised price system.*

If each consumer's preferences are independent of the identity of other consumers then the price space may be constrained to one of personalised prices without lump sum transfers. Less restrictively:

a.9S at any weakly efficient allocation $((\hat{x}^i_{\hat{S}}), \hat{y}_{\hat{S}})$, $Y_S \subseteq Y_{\hat{S}}$ and for all i, $P^i_S(\hat{x}^i_{\hat{S}}) \subseteq P^i_{\hat{S}}(\hat{x}^i_{\hat{S}})$.

Assumptions $(9S)_Z$ require that the "out of equilibrium" production and Pareto preferred sets be nested with reference to the "in equilibrium" production and Pareto preferred sets.

Theorem 2 *Under* $(1S, \ldots, 9S)_Z$*, if* $((\hat{x}^i_{\hat{S}}),\ \hat{y}_{\hat{S}})$ *is weakly efficient there exists a non-negative price vector* (v^i)*, where* $(\hat{v}^i, 0)$ *is in* $\triangle$ *for every* i*, such that* $((\hat{x}^i_{\hat{S}}),\ \hat{y}_{\hat{S}},\ (\hat{v}^i))$ *is a quasi-equilibrium relative to a personalised price system without lump sum transfers.*

If, in addition, $\hat{v}^i\hat{x}^i_{\hat{S}} \neq \inf \hat{v}^i X^i_S$*, for all* i *and all* S*, then* $((\hat{x}^i_{\hat{S}}),\ \hat{y}_{\hat{S}},\ (\hat{v}^i))$ *is an equilibrium relative to a personalised price system without lump sum transfers.*

If each consumer's preferences are independent of the identity of other

[6] Consider any two vectors $x = (x^i)$ and $y = (y^i)$. If $x^i \geq y^i$ for all i then $x \geq y$, if $x^i \geq y^i$ for all i and $x^i > y^i$ for some i then $x > y$ and if $x^i > y^i$ for all i then $x \gg y$.

consumers and, in the neighbourhood of the weakly efficient allocation, are identical then the dual space may be constrained to one of non-personalised prices without lump sum transfers.

a.10 at any weakly efficient allocation $((\hat{x}^i_{\hat{S}}), \hat{y}_{\hat{S}})$, for every i and i' and some neighbourhood O of $\hat{x}^i_{\hat{S}}$ and O' of $\hat{x}^{i'}_{\hat{S}}$, $O \cap P^i_{\hat{S}}(\hat{x}^i_{\hat{S}}) = O' \cap P^{i'}_{\hat{S}}(\hat{x}^{i'}_{\hat{S}})$.

Theorem 3 *Under* $(1S, \ldots, 9S)_Z$ *and 10, if* $((\hat{x}^i_{\hat{S}}), \hat{y}_{\hat{S}})$ *is weakly efficient there exists a non-negative price vector* $\hat{v}$, *where* $(\hat{v}, 0)$ *is in* $\triangle$, *such that* $((\hat{x}^i_{\hat{S}}), \hat{y}_{\hat{S}}, \hat{v})$ *is a quasi-equilibrium relative to a non-personalised price system without lump sum transfers.*

If, in addition, $\hat{v}\hat{x}^i_{\hat{S}} \neq \inf \hat{v}X^i_S$, *for all* i *and all* S, *then* $((\hat{x}^i_{\hat{S}}), \hat{y}_{\hat{S}}, (\hat{v}))$ *is an equilibrium relative to a non-personalised price system without lump sum transfers.*

5 Existence of Equilibrium

An allocation $((\hat{x}^i_{\hat{S}}), \hat{y}_{\hat{S}})$ is a *quasi-equilibrium* at complete personalised prices $(\hat{v}^i_S)$ and lump sum transfers $(\hat{\tau}^i_S)$ if and only if it is feasible and

(1) for all S and every y_S in Y_S, $\hat{u}_{\hat{S}}\hat{y}_{\hat{S}} \geq \hat{u}_S y_S$,

(2) for all i, $\hat{v}^i_{\hat{S}}\hat{x}^i_{\hat{S}} = qw^i$, and for all S and for every x^i_S in X^i_S such that $x^i_S \succeq^i \hat{x}^i_{\hat{S}}$, $\hat{v}^i_S x^i_S + \hat{\tau}^i_S \geq \hat{v}^i_{\hat{S}}\hat{x}^i_{\hat{S}} + \hat{\tau}^i_{\hat{S}}$.

Quasi-equilibrium and equilibrium at *personalised prices* and *non-personalised prices* are defined analogously.

If (1) and (2′) hold and $((\hat{x}^i_{\hat{S}}), \hat{y}_{\hat{S}})$ is feasible then it is an *equilibrium at a complete personalised price system.*

(2′) for all i, $\hat{v}^i_{\hat{S}}\hat{x}^i_{\hat{S}} = qw^i$, and for all S and for every x^i_S in X^i_S such that $x^i_S \succ^i \hat{x}^i_{\hat{S}}$, $\hat{v}^i_S x^i_S + \hat{\tau}^i_S > \hat{v}^i_{\hat{S}}\hat{x}^i_{\hat{S}} + \hat{\tau}^i_{\hat{S}}$.

Equilibrium at *personalised prices* and *non-personalised prices* are defined analogously.

Generally the proof of existence for the class of models presented in Section 4 is difficult and beyond the scope of this paper. Commodities are indexed by the partition with which they are associated. Production may never occur under more than one partition. This generates discontinuities in the demand and supply correspondences as prices change. As prices change consumers may migrate from one region to another. As a consumer migrates his (her) demand correspondence in his (her) former residence takes on the value zero. Therefore the aggregate demand and supply correspondences under the former partition take on the value zero.

However, a technique for avoiding such discontinuities suggests itself. To prove the existence of an equilibrium at complete personalised prices, given the Second Welfare Theorem holds, it is sufficient to prove that for some Pareto optimal (and so weakly efficient) allocation $(\hat{x}^i_S)$ for every i, $\hat{v}^i_{\hat{S}}\hat{x}^i_{\hat{S}} = qw^i$ (5.1). The sequence of complete personalised prices $(\hat{v}^i_S)$ and lump sum transfers $(\hat{\tau}^i_S)$ is a selection from the set of complete personalised prices that support the Pareto optimal allocation $(\hat{x}^i_{\hat{S}})$.

If some Pareto optimal allocations are associated with a partition different from other Pareto optimal allocations then the search for the Pareto optimal allocation that satisfies equation (5.1) must be conducted under more than one partition. This would necessitate accounting for discontinuities in the supply, demand and price correspondences as the search for the appropriate Pareto optimum moved from one partition to another.

An alternative characterisation of why it is difficult to prove the existence of an equilibrium in local public good models is to be found in Diamantaras and Wilkie (1992, Theorem 6, p. 9). Diamantaras and Wilkie prove that the set of Pareto optimal allocations in local public good models need not be connected. This implies that, in general, no fixed point theorem can be applied to the set of Pareto optimal allocations to obtain an allocation such that each consumers budget constraint is satisfied.

If all Pareto optimal allocations are associated with one partition the discontinuities associated with the search for a Pareto optimal allocation that satisfies equation (5.1) can be avoided. In fact, once the partition that has associated with it all Pareto optimal allocations is identified, the Pareto optimal allocation(s) that satisfy equation (5.1) can be found through the application of a standard existence result. This is done in Theorems 4, 5 and 6.

a.11 $\hat{S}$ is unique,
a.12 for every i, $X^i_{\hat{S}}$ bounded below in $\leq$,[7]
a.13 for every i, $X^i_{\hat{S}}$ closed,

[7] That $X^i_{\hat{S}}$ is bounded below means there is a point χ^i in $\Re^{GJ+M}$ such that $\chi^i \leq x^i$ for all x^i in $X^i_{\hat{S}}$.

a.14 the relative interiors of $Y_{\hat{S}}$ and $X_{\hat{S}}$ have a non-empty intersection,
a.15 $Y_{\hat{S}}$ is closed,
a.16 $Y_{\hat{S}} \cap \Re_+^{GJ+M} = \{0\}$.

Theorem 4 *Under* $(1S, \ldots, 8S)_{Z\setminus\hat{S}}$ *and* 11 *through* 16 *there exists an allocation* $((\hat{x}^i_{\hat{S}}), \hat{y}_{\hat{S}})$ *that is a quasi-equilibrium at complete personalized prices* $(\hat{v}^i_S)$ *and lump sum transfers* $(\hat{\tau}^i_S)$*, where* $(\hat{v}^i_S, \hat{\tau}^i_S)$ *is in* $\triangle$ *for every* i*,* $\hat{\tau}^i_{\hat{S}} = 0$ *for all* i *and* $\sum_i \hat{\tau}^i_S = 0$ *for all* S*, with non-negative public and private goods prices.*

If, in addition, $\hat{v}^i_{\hat{S}}\hat{x}^i_{\hat{S}} \neq \inf \hat{v}^i_S X^i_S$*, for all* i *and all* S *then* $((\hat{x}^i_{\hat{S}}), \hat{y}_{\hat{S}}, (\hat{v}^i_{\hat{S}}), (\hat{\tau}^i_{\hat{S}}))$ *is an equilibrium at complete personalized prices.*

Theorem 5 *Under* $(1S, \ldots, 9S)_{Z\setminus\hat{S}}$ *and* 11 *through* 16 *there exists an allocation* $((\hat{x}^i_{\hat{S}}), \hat{y}_{\hat{S}})$ *that is a quasi-equilibrium at non-negative personalised prices* $(\hat{v}^i)$*, where* $(\hat{v}^i_S, 0)$ *is in* $\triangle$ *for every* i*, without lump sum transfers.*

If, in addition, $\hat{v}^i_{\hat{S}}\hat{x}^i_{\hat{S}} \neq \inf \hat{v}^i_S X^i_S$*, for all* i *and all* S *then* $((\hat{x}^i_{\hat{S}}), \hat{y}_{\hat{S}}, (\hat{v}^i))$ *is an equilibrium at personalized prices, without lump sum transfers.*

Theorem 6 *Under* $(1S, \ldots, 9S)_{Z\setminus\hat{S}}$*, and* 10 *through* 16 *there exists an allocation* $((\hat{x}^i_{\hat{S}}), \hat{y}_{\hat{S}})$ *that is a quasi-equilibrium at non-negative non-personalised prices* $(\hat{v})$*, where* $(\hat{v}, 0)$ *is in* $\triangle$*, without lump sum transfers.*

If, in addition, $\hat{v}^i_{\hat{S}}\hat{x}^i_{\hat{S}} \neq \inf \hat{v}^i_S X^i_S$*, for all* i *and all* S *then* $((\hat{x}^i_{\hat{S}}), \hat{y}_{\hat{S}}, (\hat{v}))$ *is an equilibrium at non-personalized prices, without lump sum transfers.*

Among the class of models to which Theorem 4 applies the following subclasses may generate Pareto optimal allocations that are all associated with one partition, the class of *pure public good* models (as in Foley (1970)), some of the class of *pure public service* models with at least as many regions as preference types (as in Bewley (1981, Section 9, p. 729)) and models with *non-anonymous crowding in production.*

6 The Core

In this section we extend the definition of the Core introduced by Foley (1970) to economies with local public goods. The notion of the core we introduce incorporates the idea that all defecting coalitions can by some means exclude others from enjoying the local public goods they produce. We allow defecting coalitions $\mathcal{C}$ to form any number of regions they wish, $J_{\mathcal{C}}$ say. We use the convention $\mathcal{C} = \{1, \ldots, c, \ldots, C\}$ and similarly for $\mathcal{J}_\mathcal{C}$.

Consumer residence choice when a member of the defecting coalition $\mathcal{C} \subset \mathcal{I}$ is indicated by the partition of $\mathcal{C}$, $S_\mathcal{C}$, where $\#S_\mathcal{C} = J_\mathcal{C}$. In this section it is allowed that any partition of all consumer in I, S, may be into any number of regions.

Let $\underline{g}^k = (0, \ldots, g^k, \ldots, 0) \in \Re^{GJ_\mathcal{C}}$. The consumption vector of consumer i when residing in region k as a member of the defecting coalition $\mathcal{C}$ is $x^i_{S_\mathcal{C}} = (\underline{g}^k_{S_\mathcal{C}}, l^i_{S_\mathcal{C}})$, where $l^i_{S_\mathcal{C}}$ is the consumption vector of private goods and $\underline{g}^k_{S_\mathcal{C}}$ is the consumption vector of public goods relative to the partition $S_\mathcal{C}$. The aggregate allocation is $x_{S_\mathcal{C}} = (g_{S_\mathcal{C}}, l_{S_\mathcal{C}})$, where $g_{S_\mathcal{C}} = (g^k_{S_\mathcal{C}})$ is the consumption of local public goods by all consumers in the defecting coalition $\mathcal{C}$ and $l_{S_\mathcal{C}} = \sum_i l^i_{S_\mathcal{C}}$ is the aggregate consumption of private goods.

Each consumer i has a consumption set in the space of local public goods and private goods relative to the partition $S_\mathcal{C}$, $X^i_{S_\mathcal{C}} \subset \Re^{GJ_\mathcal{C}+M}$. The preferences of consumer i over the consumption set of consumer i are represented by a complete preordering $\succ^i$ over X^i.

The set of production opportunities in the space of local public goods and private goods relative to the partition of consumers in the defecting coalition $S_\mathcal{C}$ is $Y_{S_\mathcal{C}} \subset \Re^{GJ_\mathcal{C}+M}$. The net output of local public goods and private goods relative to the partition $S_\mathcal{C}$ is $y_{S_\mathcal{C}}$.

It is maintained in this section that there are no increasing returns to coalition size (NIRCS). Defecting coalitions do not have access to a technology superior to the grand coalition. Let $\mathcal{D}$ be any subset of $\mathcal{I}, \mathcal{D} \subseteq \mathcal{I}$.

(NIRCS) *Consider any defecting coalition $\mathcal{C} \subseteq \mathcal{D}$, their allocation amongst $J_\mathcal{C}$ regions and the set of production opportunities defined with respect to the partition of $\mathcal{C}$, $Y_{S_\mathcal{C}} \subset \mathcal{R}^{GJ_\mathcal{C}+M}$. Consider the residual population $\mathcal{D}\backslash\mathcal{C}$ and the set of production opportunities defined with respect to any allocation of these consumers amongst $J_{\mathcal{D}\backslash\mathcal{C}}$ regions and the set of production opportunities defined with respect to the partition of $\mathcal{D}\backslash\mathcal{C}$, $Y_{S_{\mathcal{D}\backslash\mathcal{C}}} \subset \mathcal{R}^{GJ_{\mathcal{D}\backslash\mathcal{C}}+M}$. Denote the embeddings[8] of $Y_{S_\mathcal{C}}$ and $Y_{S_{\mathcal{D}\backslash\mathcal{C}}}$ in $\mathcal{R}^{G(J_\mathcal{C}+J_{\mathcal{D}\backslash\mathcal{C}})+M}$ by $Y^*_{S_\mathcal{C}}$ and $Y^*_{S_{\mathcal{D}\backslash\mathcal{C}}}$ respectively. Then*

$$Y^*_{S_\mathcal{C}} + Y^*_{S_{\mathcal{D}\backslash\mathcal{C}}} \subseteq Y^*_{S_\mathcal{D}}.$$

[8] *Consider $A \subseteq R^c$ and R^d, where $d > c$. We say the embedding of A in R^c is a set $A^* \subset R^d$ such that $a \in A \Leftrightarrow a^* \in A^*$ where a^* is the vector $a^* = (a, 0)$ where 0 is a $d - c$ dimensional vector.*

Intuitively, no coalitions C and $D\backslash C$ can produce an allocation that their union cannot. In particular, no coalitions C and $I\backslash C$ can produce an allocation that the grand coalition cannot.

An allocation $((x^{\star i}_{S^\star_{\mathcal{C}}}), y^\star_{S^\star_{\mathcal{C}}})$ is C-*feasible* if

(1) $y^\star_{S^\star_{\mathcal{C}}}$ is in $Y_{S^\star_{\mathcal{C}}}$ and for all i, $x^{\star i}_{S^\star_{\mathcal{C}}}$ is in $X^i_{S^\star_{\mathcal{C}}}$,
(2) $y^\star_{S^\star_{\mathcal{C}}} + \sum_{\mathcal{C}} w^i = x^\star_{S^\star_{\mathcal{C}}}$.

An allocation $((x^{\star i}_{S^\star}), y^\star_{S^\star})$ is *globally weakly efficient* if

(3) if there exists an allocation $((x'^i_{S'}), y'_{S'})$ such that $x'^i_{S'} \succ^i x^{\star i}_{S^\star}$ for all i then $((x'^i_{S'}), y'_{S'})$ is not feasible.

An allocation is globally weakly efficient if there exists no other feasible allocation that is better for all consumers under any partition of consumers into any number of regions J in $\mathcal{N}$, (where $\mathcal{N}$ is the set of natural numbers).

An allocation $((x^{\star i}_{S^\star}), y^\star_{S^\star})$ is *blocked* by a coalition $\mathcal{C} \neq \emptyset$ if there exists a C-feasible allocation $((x'^i_{S'_{\mathcal{C}}}), y'_{S'_{\mathcal{C}}})$ such that

(4) $x'^i_{S'_{\mathcal{C}}} \succeq^i x^{\star i}_{S^\star_{\mathcal{C}}}$ for all i in $\mathcal{C}$ and $x'^i_{S'_{\mathcal{C}}} \succ^i x^{\star i}_{S^\star_{\mathcal{C}}}$ for some i in $\mathcal{C}$.

An allocation is in the *core* if it cannot be blocked. Assume the following for each defecting coalition $\mathcal{C}$.

It is useful to introduce that notion of an "extended" quasi-equilibrium. An allocation $((\hat{x}^i_{\hat{S}}), \hat{y}_{\hat{S}})$ is an *extended quasi-equilibrium* at complete personalised prices $\left((\hat{v}^i_S)_{Z_{\mathcal{C}}}\right)_{\mathcal{C}\subseteq\mathcal{I}}$ and lump sum transfers $\left((\hat{\tau}^i_S)_{Z_{\mathcal{C}}}\right)_{\mathcal{C}\subseteq\mathcal{I}}$ if and only if it is (1), (2), (3′) and (4′) hold.

(3′) for all $\mathcal{C} \subset \mathcal{I}$, for all $S_{\mathcal{C}}$ in $Z_{\mathcal{C}}$ and every $y_{S_{\mathcal{C}}}$ in $Y_{S_{\mathcal{C}}}$, $\hat{u}_{\hat{S}}\hat{y}_{\hat{S}} \geq \hat{u}_{S_{\mathcal{C}}} y_{S_{\mathcal{C}}}$,
(4′) for all i, $\hat{v}^i_{\hat{S}}\hat{x}^i_{\hat{S}} = qw^i$, and for all $\mathcal{C} \subset \mathcal{I}$, for all $S_{\mathcal{C}}$ in $Z_{\mathcal{C}}$ and for every $x^i_{S_{\mathcal{C}}}$ in $X^i_{S_{\mathcal{C}}}$ such that $x^i_{S_{\mathcal{C}}} \succeq^i \hat{x}^i_{\hat{S}}$, $\hat{v}^i_{S_{\mathcal{C}}} x^i_{S_{\mathcal{C}}} + \hat{\tau}^i_{S_{\mathcal{C}}} \geq \hat{v}^i_{\hat{S}}\hat{x}^i_{\hat{S}} + \hat{\tau}^i_{\hat{S}}$.

Extended quasi-equilibrium and equilibrium at *personalised prices* and *non-personalised prices* are defined analogously.

If (1), (2), (3′) and (4″) hold and $((\hat{x}^i_{\hat{S}}), \hat{y}_{\hat{S}})$ is feasible then it is an *extended equilibrium at a complete personalised price system.*

(4″) for all i, $\hat{v}^i_{\hat{S}}\hat{x}^i_{\hat{S}} = qw^i$, and for all $\mathcal{C} \subset \mathcal{I}$, for all $S_{\mathcal{C}}$ in $Z_{\mathcal{C}}$ and for every $x^i_{S_{\mathcal{C}}}$ in $X^i_{S_{\mathcal{C}}}$ such that $x^i_{S_{\mathcal{C}}} \succ^i \hat{x}^i_{\hat{S}}$, $\hat{v}^i_{S_{\mathcal{C}}} x^i_{S_{\mathcal{C}}} + \hat{\tau}^i_{S_{\mathcal{C}}} > \hat{v}^i_{\hat{S}}\hat{x}^i_{\hat{S}} + \hat{\tau}^i_{\hat{S}}$.

Extended equilibrium at *personalised prices* and *non-personalised prices* are defined analogously.

Theorem 7 Under $((1S_{\mathcal{C}},\ldots,8S_{\mathcal{C}})_{Z_{\mathcal{C}}})_{\mathcal{C}\subseteq\mathcal{I}}$ *and* $11,\ldots,16$ *there exists an allocation* $((\hat{x}^i_{\hat{S}}),\hat{y}_{\hat{S}})$ *that is an extended quasi-equilibrium at complete prices* $\left((\hat{v}^i_S)_{Z_{\mathcal{C}}}\right)_{\mathcal{C}\subseteq\mathcal{I}}$ *and lump sum transfers* $\left((\hat{\tau}^i_S)_{Z_{\mathcal{C}}}\right)_{\mathcal{C}\subseteq\mathcal{I}}$*, where* $\left((\hat{v}^i_S)_{Z_{\mathcal{C}}},(\hat{\tau}^i_S)_{Z_{\mathcal{C}}}\right)_{\mathcal{C}\subseteq\mathcal{I}}$ *is in* $\triangle$ *for every* i*,* $\hat{\tau}^i_{\hat{S}}=0$ *for all* i *and* $\sum_i\hat{\tau}^i_{S_{\mathcal{C}}}=0$ *for all* $S_{\mathcal{C}}$*, with non-negative public and private goods prices.*

If, in addition, $\hat{v}^i_{\hat{S}}\hat{x}^i_{\hat{S}}\neq\inf\hat{v}^i_{\hat{S}}X^i_{\hat{S}}$*, for all* i*,* for all $\mathcal{C}\subseteq\mathcal{I}$ and for all S_C then $\left((\hat{x}^i_{\hat{S}}),\hat{y}_{\hat{S}},\left((\hat{v}^i_S)_{Z_{\mathcal{C}}}\right)_{\mathcal{C}\subseteq\mathcal{I}},\left((\hat{\tau}^i_S)_{Z_{\mathcal{C}}}\right)_{\mathcal{C}\subseteq\mathcal{I}}\right)$ *is an extended equilibium at complete personalized prices.*

a.17S$_\mathbf{C}$ for every i, $X^i_{S_{\mathcal{C}}}$ is convex,

a.18S$_\mathbf{C}$ for every i and every $x^i_{S_{\mathcal{C}}}$ in $X^i_{S_{\mathcal{C}}}$ there is a commodity bundle $x'^i_{S'_{\mathcal{C}}}$ such that $x'^i_{S'_{\mathcal{C}}}\succ^i x^i_{S_{\mathcal{C}}}$,

a.19S$_\mathbf{C}$ for every i, let $x'^i_{S_{\mathcal{C}}}$ and $x^i_{S_{\mathcal{C}}}$ be arbitrary different commodity bundles in $X^i_{S_{\mathcal{C}}}$ with $x'^i_{S'_{\mathcal{C}}}\succeq^i x^i_{S_{\mathcal{C}}}$, and $\alpha\in(0,1)$. We assume that $\alpha x'^i_{S_{\mathcal{C}}}+(1-\alpha)x^i_{S_{\mathcal{C}}}\succ^i x^i_{S_{\mathcal{C}}}$,

a.20S$_\mathbf{C}$ $Y_{S_{\mathcal{C}}}$ is a convex cone with vertex at the origin.

Theorem 8 *Under* $((17S_{\mathcal{C}},\ldots,20S_{\mathcal{C}})_{Z_{\mathcal{C}}})_{\mathcal{C}\subseteq\mathcal{I}}$ *, if* $((\hat{x}^i_{\hat{S}}),\hat{y}_{\hat{S}},(\hat{v}^i_S),(\hat{\tau}^i_S))$ *is an equilibrium at extended complete personalised prices, personalized prices or non-personalized prices then* $((\hat{x}^i_{\hat{S}}),\hat{y}_{\hat{S}})$ *is in the core.*

It follows directly from Theorem 8 that any extended equilibrium at complete personalized prices is globally weakly efficient. Under NIRCS, any equilibrium at complete personalized prices that is globally efficient is an extended equilibirum at complete personalized prices and so any equilibrium at complete personalized prices that is globally weakly efficient is in the core.

Corollary 9 *Under* $((17S_{\mathcal{C}},\ldots,20S_{\mathcal{C}})_{Z_{\mathcal{C}}})_{\mathcal{C}\subseteq\mathcal{I}}$ *, if* $((\hat{x}^i_{\hat{S}}),\hat{y}_{\hat{S}},(\hat{v}^i_S),(\hat{\tau}^i_S))$ *is globally weakly efficient and an equilibrium at complete personalised prices, personalized prices or non-personalized prices then* $((\hat{x}^i_{\hat{S}}),\hat{y}_{\hat{S}})$ *is in the core.*

7 Discussion

Since the prices that support the Pareto optimal allocations are Lindahl an incentive problem is present. If a government does not know consumer preferences then consumers may have an incentive to misrepresent their preferences so as to reduce the Lindahl price they face (see Samuelson (1954) for a discussion). Whether any equilibrium at complete personalised prices may be implemented as a Nash equilibrium is an open question.

Although the model in this paper may be interpreted as an story of how a central government, perfectly well informed about consumer preferences and production opportunities, should set prices if efficiency is desired the model may also be seen as a benchmark in the local public goods literature. The informational requirements in this paper are no more than the informational requirements in Foley (1970). Any mechanism designed to implement efficient allocations in economies with local public goods must, in the neighbourhood of that allocation, be similar to a system of complete personalized prices that would support the same efficient allocation (just as any mechanism designed to implement efficient allocations in economies with pure public goods must, in the neighbourhood of that allocation, be similar to a system of personalized prices in the sense of Foley).

Manning (1994) characterises complete personalized prices. However, characterization requires that the concept of the marginal rate of substitution and the marginal rate of transformation be generalised to allow for cases where the preference orderings or production opportunities are not smooth, that is not differentiable everywhere.

The results in this paper indicate important limitations to standard techniques for evaluating the benefits (costs) of publicly provided goods. Since Lindahl prices are required for the evaluation of the benefits of many public programs and Lindahl prices often do not exist the correct Lindahl ("shadow") prices must be constructed. We have shown that in constructing these Lindahl prices for some public good in some region information about the residence choice of all consumers and the vector of public goods provided in other regions may need to be incorporated. Theorem 2 and Theorem 3 indicate that, under restrictive conditions, those informational requirements can be relaxed and demonstrate that there are many economies for which consumers must know how the personalized prices they face for local public goods will change if they migrate for first best allocations to be supported.

8 First Appendix

Following Foley (1970), we define an artificial production set in which public commodities are treated as strictly jointly produced private commodities.

Define the production set relative to the partition S in the extended commodity space to be:

$$AY_S \equiv \{(g^1, \dots, g^I, l - w) \in \Re^{GJI+M} | g^i = \underline{g}^j \text{ for all } i \text{ in } R^j, \text{ and for all } j \text{ such that } S = (R^j) \text{ and } y_S \text{ in } Y_S\}.$$

Define the weakly better than set relative to the partition S in the extended commodity space to be:

$$AR^i_S(\hat{x}^i_{\hat{S}}) \equiv \{(0, \dots, g^i, \dots, 0, l^i) \in \Re^{GJI+M} | g^i = \underline{g}^j \text{ for all } i \text{ in } R^j, \text{and for all}$$

j such that $S = (R^j), x^i_S$ is in X^i_S and $x^i_S \succeq^i \hat{x}^i_{\hat{S}}$ for all $S\}$.

Define the strictly better than set relative to the partition S in the extended commodity space to be:

$$AP^i_S(\hat{x}^i_{\hat{S}}) \equiv \{(0, \dots, g^i, \dots, 0, l^i) \in \Re^{GJI+M} | g^i = \underline{g}^j \text{ for all } i \text{ in } R^j, \text{and for all}$$

j such that $S = (R^j), x^i_S$ is in X^i_S and $x^i_S \succ^i \hat{x}^i_{\hat{S}}$ for all $S\}$.

Often $AR^i_S(\hat{x}^i_{\hat{S}})$ will be written AR^i_S and $AP^i_S(\hat{x}^i_{\hat{S}})$ will be written AP^i_S for short. An element of AY_S is denoted ay_S. An element of AP^i_S or AR^i_S is denoted ax^i_S.

Define the price space for allocations in the extended commodity space to be:

$$A\triangle \equiv \{(ap, q, \tau) \in \Re^{GJI}_+ \times \Re^{M+I} | (ap, q, \tau) \neq 0\}.$$

An element of $A\triangle$ is (ap_S, q, τ_S). Denote (ap_S, q) by av_S, where $ap_S = (p^i_S)$ and $\tau_S = (\tau^i_S)$. Denote the aggregate endowment by aw.

The Pareto optimal allocation $((\hat{x}^i_{\hat{S}}), \hat{y}_{\hat{S}})$ is rewritten $((\widehat{ax}^i_{\hat{S}}), \widehat{ay}_{\hat{S}})$.

Define $P((\hat{x}^i_{\hat{S}})) \equiv \Pi_Z P_S((\hat{x}^i_{\hat{S}}))$ and, as above, define $AP((\hat{x}^i_{\hat{S}}))$. Define $G_S \equiv AY_S + aw$.

Where no confusion will arise we write $\hat{x}^i$ for $\hat{x}^i_{\hat{S}}$ and $\hat{x}$ for $(\hat{x}^i)$.

Remark: The proof of Theorem 1 is an adaption of the proof of Theorem 1 in Foley (1970, p. 68).

Proof of Theorem 1:

(i) Consider an arbitrary partition S and any weakly efficient allocation $((\hat{x}^i_{\hat{S}}), \hat{y}_{\hat{S}})$. The allocation $((\hat{x}^i_{\hat{S}}), \hat{y}_{\hat{S}})$ is feasible since it is weakly efficient. G_S is a convex cone (5S) and $G_S \neq \emptyset$ (6S) by assumption.

$AP_S(\hat{x})$ is convex since, for each i, $AP^i_S(\hat{x}^i)$ is convex (1S,3S). $AP_S(\hat{x}) \neq \emptyset$ by the comprehensiveness of X^i_S (1S) and monotonicity (4S).

(ii) $AP_S(\hat{x}) \cap G_S = \emptyset$ by the definition of Pareto optimality. By the Minkowski separating hyperplane theorem (see Takayama (1985), p. 44), for each S, there exists a price vector $\widehat{av}_S \neq 0$ and scalar r_S such that

(1) for every $ay_S + aw$ in G_S, $\widehat{av}_S(ay_S + aw) \leq r_S$,

(2) for every ax_S in $Cl\ AP_S$, $\widehat{av}_S ax_S \geq r_S$,

where $Cl\ AP_S$ is the closure of $AP_S(\hat{x})$.

(iii) Since $0 \in AY_S - aw$ (6S), $r_S \geq \widehat{av}_S aw$ for all S. If there exists in AY_S an activity ay_S such that $\widehat{av}_S(ay_S + aw) > \widehat{av}_S aw$ then that activity could be expanded indefinitely giving an unbounded profit (by constant returns to scale (5S)) and contradicting (1) of Step (ii). Therefore, we can pick $r_S = \widehat{av}_S aw$ for all S. Further, by the invariance of private goods prices to partition we can pick r_S to be independent of partition. Let $r_S = r$ for every S.

(iv) Suppose the public or private goods prices in av_S had some negative component. Points with very large amounts of the corresponding commodity would be in $Cl\ AP_S$ by the comprehensiveness of X^i_S ($1S$) and monotonicity ($4S$), but would have negative value which contradicts (2) of Step (ii). Therefore, $\widehat{av}_S > 0$.

(v) Suppose that $(q^m) = (0)$. Some $p^{gji}_S > 0$. Since production of all public goods is possible with no public good input (8S), there would be a point in AY_S with positive profit (7S), which contradicts (2) of Step (ii). Therefore, $q^m > 0$.

(vi) By feasibility $\widehat{av}_{\hat{S}} \sum_i \widehat{ax}^i_{\hat{S}} = r$. By lower semi-continuity of consumer preferences (2S) $\widehat{ax}^i_{\hat{S}}$ is the limit of points in $Cl\ AP^i_{\hat{S}}$. By Debreu (1959, 1.9 (1), p. 20) $\sum_i \widehat{ax}^i_{\hat{S}}$ is an element of $Cl\ AP_{\hat{S}}$. Therefore, condition (1) of a quasi-equilibrium relative to a complete personalised price system is satisfied.

(vii) Claim: $\sum_{\mathcal{I}} AR^i_S(\hat{x}^i) \subseteq ClAP_S(\hat{x}^i)$ That $AR^i_S(\hat{x}^i) = ClAP^i_S(\hat{x}^i)$ follows immediately by (2S) and (4S). That $\sum_{\mathcal{I}} ClAP^i_S \subseteq ClAP_S$ follows from

Debreu (1959, 1.9 (1), p.20). The claim is immediate.

Define $ax'^k_{\hat{S}}$ to be such that $x'^k_{\hat{S}} \succeq^k \hat{x}^k_{\hat{S}}$. Is has been demonstrated that $\widehat{av}_{\hat{S}} \sum_{\mathcal{I}\backslash k} \widehat{ax}^i_{\hat{S}} + \widehat{av}_{\hat{S}} ax'^k_{\hat{S}} \geq \widehat{av}_{\hat{S}} \sum_i \widehat{ax}^i_{\hat{S}}$ and so it follows immediately that $\widehat{av}_{\hat{S}} ax'^k_{\hat{S}} \geq \widehat{av}_{\hat{S}} \widehat{ax}^k_{\hat{S}}$. Define $(ax'^i_{S'})$ to be such that $x'^i_{S'} \succeq^i \hat{x}^i_{\hat{S}}$ for all i in $\mathcal{I}$. It has been demonstrated that $\widehat{av}_{S'} \sum_i ax'^i_{S'} \geq \widehat{av}_{\hat{S}} \sum_i \widehat{ax}^i_{\hat{S}}$. To show that $\widehat{av}_{S'} ax'^i_{S'} \geq \widehat{av}_{\hat{S}} \widehat{ax}^i_{\hat{S}}$, for all i in $\mathcal{I}$ note that net payments τ^i_S, to any consumer and under any partition, may be of any sign. Therefore condition (2) of a quasi-equilibrium relative to a complete personalised price system is satisfied.

(viii) Consider $\widehat{av}_{S'} ax'^i_{S'} \geq \widehat{av}_{\hat{S}} \widehat{ax}^i_{\hat{S}}$. Suppose $x'^k_{\hat{S}} \succ^k \hat{x}^k_{\hat{S}}$ and equality held. By assumption, there is a point with lower value than $\widehat{av}_{S'} ax'^i_{S'}$ under each partition S' in Z. Along the line between this point and $ax'^i_{S'}$ all points have lower value than $\widehat{ax}^i_{\hat{S}}$, but near $ax'^i_{S'}$, by lower semi-continuity (2S), there will be a point preferred to $\widehat{ax}^i_{\hat{S}}$. Therefore, there is a point in $Cl\ AP_{S'}$ with value less than the endowment, which contradicts (2) of Step (ii). Therefore $\widehat{av}_{S'} ax'^i_{S'} > \widehat{av}_{\hat{S}} \widehat{ax}^i_{\hat{S}}$ and condition (2′) of an equilibrium relative to a complete personalised price system is satisfied. *Q.E.D.*

Proof of Theorem 2:

(i) is as in the proof of Theorem 1.

(ii) is as in the proof of Theorem 1 but add "By (10) we can pick $\widehat{av}_S = \widehat{av}_{\hat{S}}$, for all S. Let $\widehat{av}_{\hat{S}} = \widehat{av}$."

(iii), (iv), (v) and *(vi)* are as in the proof of Theorem 1. In each case replace $\widehat{av}_S$ with $\widehat{av}$ for all S.

(vii) Assume $ax'^k_{\hat{S}} = \widehat{ax}^k_{\hat{S}}$, for all $k \neq i$. Then $\widehat{av} ax'^i_{\hat{S}} \geq \widehat{av}\widehat{ax}^i_{\hat{S}}$. Define $ax'^k_{S'}$, $S' \neq \hat{S}$, to be such that $ax'^k_{S'} \succ^k \widehat{ax}^k_{\hat{S}}$. By (9S) $\widehat{av} ax'^i_{S'} \geq \widehat{av}\widehat{ax}^i_{\hat{S}}$.

Therefore condition (2) of a quasi-equilibrium relative to a personalised price system is satisfied.

(viii) As in the proof of Theorem 1 with $\widehat{av}_S$ replaced by $\widehat{av}$ for all S. *Q.E.D.*

Let the neighbourhood of $\widehat{ax}^k_{\hat{S}}$ in $\Re^{GJI+M}$ be O.

Proof of Theorem 3:

(i) is as in the proof of Theorem 1.

(ii) is as in the proof of Theorem 2.

(iii), (iv), (v) and *(vi)* are as in the proof of Theorem 1. In each case replace $\widehat{av}_S$ with $\widehat{av}$ for all S.

(vii) Assume $ax'^k_{\hat{S}} = \widehat{ax}^k_{\hat{S}}$, for all $k \neq i$. Then $\widehat{av} ax'^i_{\hat{S}} \geq \widehat{av}\widehat{ax}^i_{\hat{S}}$. Define

$ax'^k_{S'}$, $S' \neq \hat{S}$, to be such that $ax'^k_{S'} \succ^k \widehat{ax}^k_{\hat{S}}$. By $(9S)$ $\widehat{av}ax'^i_{S'} \geq \widehat{av}\widehat{ax}^i_{\hat{S}}$.

(viii) By $(10S)$, $\widehat{av}$ may be restricted to a vector of form $\widehat{av} = ((p^g_{\hat{S}}), (q^m))$, to obtain, if $ax'^k_{\hat{S}}$ is in $O \cap P^k_{\hat{S}}(\hat{x}^k_{\hat{S}})$ then $\widehat{av}ax'^i_{\hat{S}} \geq \widehat{av}\widehat{ax}^i_{\hat{S}}$. To show that, if $ax'^k_{\hat{S}}$ is in $AP^k_{\hat{S}}(\hat{x}^k_{\hat{S}})$, then $\widehat{av}ax'^i_{\hat{S}} \geq \widehat{av}\widehat{ax}^i_{\hat{S}}$, note that $AP^k_{\hat{S}}(\hat{x}^k_{\hat{S}})$ is convex. Let H be the hyperplane, through $\widehat{ax}^k_{\hat{S}}$, that supports $O \cap AP^k_{\hat{S}}(\hat{x}^k_{\hat{S}})$. Let H not support $AP^k_{\hat{S}}(\hat{x}^k_{\hat{S}})$ so $H \cap AP^k_{\hat{S}}(\hat{x}^k_{\hat{S}}) \neq \emptyset$. By the convexity of $AP^k_{\hat{S}}(\hat{x}^k_{\hat{S}})$, $H \cap O \cap AP^k_{\hat{S}}(\hat{x}^k_{\hat{S}}) \neq \emptyset$, a contradiction.

(ix) As in (viii) of the proof of Theorem 1 with $\widehat{av}_S$ replaced by $\widehat{av}$ for all S. *Q.E.D.*

Proof of Theorem 4: By $(1S, \ldots, 8S)_Z$ any Pareto optimal allocation $((\hat{x}^i_{\hat{S}}), \hat{y}_{\hat{S}})$ has a supporting price, with no lump sum transfers under the partition $\hat{S}$. Let the supporting price under each partition S be in the set $\triangle^{L,\tau=0}_S((\hat{x}^i_{\hat{S}}), \hat{y}_{\hat{S}})$. Since private goods prices are invariant to the partition and migration only is possible $(6S)_Z$, each consumer's wealth is invariant to the partition with which his (her) residence is associated.

Therefore, it is sufficient to demonstrate that there exists a Pareto optimal allocation $(\hat{x}^i_{\hat{S}})$ and a supporting price $(\hat{v}^i_{\hat{S}})$ in $\triangle^{L,\tau=0}_{\hat{S}}((\hat{x}^i_{\hat{S}}), \hat{y}_{\hat{S}})$ such that for every i, $\hat{v}^i_{\hat{S}}\hat{x}^i_{\hat{S}} = qw^i$.

By assumption 11 any Pareto optimal allocation is associated with a unique partition $\hat{S}$. Therefore, to prove the existence of an equilibrium at personalised prices it is sufficient to prove the existence of an equilibrium under the partition $\hat{S}$.

By the Proposition in Appendix II there exists an equilibrium under the partition $\hat{S}$. To see this 12 implies (0), $1\hat{S}$ and 13 imply (1), $1\hat{S}$ and $4\hat{S}$ imply (2), $2\hat{S}$ implies (3), (4) is implied by $2\hat{S}$ and $3\hat{S}$ (see (1) of 4.7 from Debreu (1959, p.60)), 14 implies (5), $5\hat{S}$ and 15 imply (6) and 16 implies (7). By the Theorem in Appendix III the equilibrium under the partition $\hat{S}$ is Pareto optimal. To see this $1\hat{S}$ implies (8); $3\hat{S}$ implies (9) and $1\hat{S}$ and $4\hat{S}$ imply (10). Therefore, there exists a Pareto optimal allocation $(\hat{x}^i_{\hat{S}})$ and a supporting price $(\hat{v}^i_{\hat{S}})$ in $\triangle^{L,\tau=0}_{\hat{S}}((\hat{x}^i_{\hat{S}}), \hat{y}_{\hat{S}})$ such that for every i, $\hat{v}^i_{\hat{S}}\hat{x}^i_{\hat{S}} = qw^i$ (5.2).

By equation 5.2 and Theorem 2 an equilibrium at non-negative personalised prices exists. *Q.E.D.*

Proof of Theorem 5: Immediate.

Proof of Theorem 6: Immediate.

Proof of Theorem 7: Immediate by application of the steps in Theorem 1 and Theorem 4 and noting that if $((\hat{x}^i_{\hat{S}}), \hat{y}_{\hat{S}})$ is globally weakly efficient then $AP_{S_\mathcal{C}}(\hat{x}) \cap G_{S_\mathcal{C}} = \emptyset$ for every $S_\mathcal{C}$ in $Z_\mathcal{C}$.

Proof of Theorem 8: Let $((\hat{x}^i_{\hat{S}}), \hat{y}_{\hat{S}})$ be an extended equilibrium at complete personalised prices, personalised prices or non-personalised prices. Suppose that $((\hat{x}^i_{\hat{S}}),\ \hat{y}_{\hat{S}})$ can be blocked by a coalition $\mathcal{C}$ with an allocation $((x^i_{S_\mathcal{C}}),\ y_{S_\mathcal{C}})$. By 20$\hat{S}$, $\hat{u}_{\hat{S}}\hat{y}_{\hat{S}} = 0$. By the Lemma, $x^i_{S_\mathcal{C}} \succ^i \hat{x}^i_{\hat{S}}$ implies $\hat{v}^i_{S_\mathcal{C}} x^i_{S_\mathcal{C}} > qw^i$.

That $((x^i_{S_\mathcal{C}}), y_{S_\mathcal{C}})$ is C-feasible implies $\sum_\mathcal{C}(x^i_{S_\mathcal{C}} - w^i) = y_{S_\mathcal{C}}$ where $y_{S_\mathcal{C}}$ is in $Y_{S_\mathcal{C}}$. Since $\hat{v}^i_{S_\mathcal{C}} x^i_{S_\mathcal{C}} \geq qw^i$ for all i with strict inequality for one i implies $\sum_\mathcal{C} \hat{v}^i_{S_\mathcal{C}} x^i_{S_\mathcal{C}} > \sum_\mathcal{C} qw^i$, or $\hat{u}_{S_\mathcal{C}} y_{S_\mathcal{C}} > 0$, a contradiction. *Q.E.D.*

Proof of Corollary 9: Immediate.

9 Second Appendix

Consider an economy with I consumers, n private commodities and one firm. There are no regions or public commodities. Let the preference preordering of the ith consumer be represented by $\succeq^i$ and let his (her) initial endowment be w^i, for $i = 1, \ldots, I$. Y denotes the production possibility set of the unique firm. Supply is assumed constant returns to scale and competitive so the firm earns no profits. From Debreu (1962) a *quasi-equilibrium* of the economy $\xi = ((X^i, \succeq^i), (Y), (w^i))$ is an $(I+2)-$tuple $((x^{\star i}), (y^\star), p^\star)$ of $((X^i), (Y), \Re^n)$, respectively, such that

(α) for every i, $x^{\star i}$ is the greatest element of $\{x^i \in X^i | p^\star x^i \leq p^\star w^i\}$ for $\succeq^i$,
(β) $p^\star y^\star = Max\ p^\star Y$,
(γ) $\sum_i x^{\star i} - y^\star = \sum_i w^i$,
(δ) $p^\star \neq 0$.

The following adaption of Milleron's (1972) assumptions and proof (see Milleron (1972, p. 443, Theorem 3.1 and p. 445)) and Debreu's (1962) equilibrium existence theorem is used to select the Pareto optimal allocations that satisfy (1). Here, unlike Debreu (1962) and Milleron (1972), Y is the set of gross outputs.

Theorem 1: *The economy ξ has a quasi-equilibrium if,*

(0) *for every i, X^i is bounded below in $\leq$,*

(1) *for every i, X^i is closed and convex,*

(2) *for every i, for every $\hat{x}^i$ in $\hat{X}^i$ there is an x^i in X^i preferred to $\hat{x}^i$,*

(3) *preferences are continuous,*

(4) *preferences are weakly convex,*

(5) *the relative interiors of Y and X have a non-empty intersection,*

(6) *$Y - w$ is a closed and convex cone,*

(7) $(Y - w) \cap \Re^n_+ = \{0\}$.

Proof: Theorem 1 is proved by showing that assumptions $(0), \ldots, (8)$ imply Debreu's assumptions (1962, p. 260 and p. 270).

(0) implies $a.1$, (1) implies $a.2$, (3) implies $b.2$ and (4) implies $b.3$. For establishing $b.1$ it is sufficient to notice that, if one defines $\hat{X}^i$ as Debreu (1959) does, (2) implies $b.1$. (5) implies c. (6) implies $d.1$ and (7) implies $d.2$. *Q.E.D.*

Consider the economy described above. The following adaption of Debreu's (1959, p. 94) First Welfare Theorem is used to ensure that the selection made using Theorem 2 is Pareto optimal.

Theorem 2: *An equilibrium $((x^{\star i}), (y^\star), (p^\star))$ is Pareto optimal if,*

(8) *for every i, X^i is convex,*

(9) *for every i, if x^i_1 and x^i_2 are two points of X^i and if $t \in (0, 1)$, then $x^i_1 \succ^i x^i_2$ implies $tx^i_1 + (1 - t)x^i_2 \succ^i x^i_1$,*

(10) *for every i, if $\sum_i x^{\star i}$ is in Y then $P^i(x^{\star i}) \neq \emptyset$.*

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