Existence of Competitive Equilibrium with a system of Complete Prices*

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Abstract

For a model with private commodities, where consumer choice over ørms is discrete, we prove that for any Pareto optimal allocation, there exists a system of complete prices and lump sum transfers between consumers and ørms that will support that Pareto optimal allocation as an equilibrium. The form of complete prices required depends upon how each consumers preferences change as any consumer ımigratesȷ between ørms. The law of one price must, in general, be violated to support Pareto optimal allocations. Selections from the set of Pareto optimal allocations are made to prove the existence of an equilibrium.

Key words: private commodities, complete prices, non-anonymous crowding in consumption, non-anonymous crowding in production

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1. Introduction

Non-convexities are ubiquitous. The literature on general equilibrium has developed techniques, in part, to avoid non-convexities. Among these techniques include replicating an economy with a ønite number of consumers and commodities, considering a continuum of consumers, allowing for an inønite number of consumers and/or commodities and endowing consumers with the ability to randomise.

In this paper we address existence of equilibrium for a class of general equilibrium models with non-convexities and private commodities. The non-convexities here are generated by discreteness in consumer choice. It is in the nature of some commodities that they may only be purchased from one ørm although many ørms may produce the commodity. The analogy with location choice is clear. Each ørm may be thought of as occupying a diœerent location. An example is ski øelds[1] and another is restaurant meals. Neither of these commodities may be considered to be inønitely divisible or even partly divisible. You either spend one day skiing on one mountain or another mountain. it is not possible to ski at both ski øelds in the same day.

The analogy with location choice can be taken much further. Hotelling has a model that can be stylised as follows: there exists a ılinear cityȷ with a continuum of consumers distributed uniformely on the interval $[0,1]$. Each consumer has unit demand. There are two ørms or stores which sell the same physical commodity located at the extremes of the city; store 1 is at $x=0$ and store 2 is at $x=1$. Consumers can go to one or the other store. Hotelling assumes the unit cost of each store is c. Hotelling addresses the question of the nature of demand in the face of transportation costs. A positive model of strategic behaviour between the two ørms is developed. Suppose that marginal cost of each store is increasing in output. Suppose that the number of consumers is ønite. Assume that transportation costs are zero. This stylization of Hotelling's model coincides with ours.

A slightly more elaborate example is as follows: Consider a large country, say Japan, looking to sign a defence pact with one of two opposing world military powers, say the United States and China. Japan has no army (let us suppose its

[1] See Barro and Romer (1987). Barro and Romer allow consumers to be ıfractionatedȷ between ørms but disallow consumers to ski at more than one ski øeld.

constitution does not allow its own nationals to participate in an armed force). It must lease the professional military services of one country or another. Allocating more men and women to military service in the United States or China can only be done at increasing marginal cost. If Japan switches pact partners then that must change the shadow price of military services in both countries. Generally, the shadow price of military services in both countries may diœer.

The question of when equilibria exist with discreteness of consumer choice has been addressed in the literature on local public goods by Wooders (1989), Greenberg (1983) and Manning (1993b) among many others. In each of these models existence is proved for a class of models with local public goods. Consumers may only consume local public goods produced by one region. Wooders avoids the non-convexities generated by the discreteness in consumer location choice by replicating a model with a ønite number of consumers and commodities. Greenberg avoids the non-convexities generated by the discreteness in consumer location choice by considering a continuum of consumers. Manning addresses the non-convexities directly by considering a model with a ønite number of consumers and commodities, indexing public commodities and personalized prices of public commodities by the allocation of consumers among regions.

In this paper, private commodities and the price of private commodities are indexed by the allocation of consumers among ørms.

Two candidate price spaces are considered. Each is associated with a diœerent characterisation of the commodity space.

In the ørst price space each consumer is charged the same price for each commodity, whatever his (her) choice of ørm or the choice of ørm of other consumers. The second price space is characterised by each consumer being charged a price for each commodity that varies as the consumers' ørm of choice varies. Further, the price each consumer faces may vary as the ørm of choice of any other consumer varies.

The class of commodities I introduce allows for any form of congestion in production or consumption. For instance, the utility I attain by eating at a restaurant may be increased if my girlfriend has a meal with me in the same restaurant. On the other hand the presence of my girlfriend in the restaurant may distract the waiter and lower his productivity. In general, the identity of consumers is treated as a local public good that anybody purchasing private commodities from the same ørm can enjoy.

The model is presented in Section 2. An example of a model that illustrates the importance of complete prices in supporting ørst best allocations is presented in Section 3. First and Second Welfare theorems are presented in Section 4 and an existence result is presented in Section 5. All proofs are presented in an Appendix.

2. The Model

We consider an economy with M private commodities, I consumers, F ørms and a Social Planner. The Social Planner is assumed to know consumer preferences and the production opportunities of each ørm. We use the convention $\mathcal{M} = \{1, \ldots, m, \ldots, M\}$ and similarly for $\mathcal{I}$ and $\mathcal{F}$.

2.1. Consumers

There are two classes of private commodities. The ørst class of commodities may be purchased by any consumer from any number of ørms in any combination and is denoted by $\mathcal{M}_1$where $\mathcal{M}_1 = \{1, \ldots, m_1, \ldots, M_1\}$. The second class of commodities may be purchased by any consumer but each consumer may only purchase from one ørm and is denoted by $\mathcal{M}_2$where $\mathcal{M}_2 = \{M_1 + 1, \ldots, m_2, \ldots, M\}$.

For each commodity m_2 in $\mathcal{M}_2$ there is a correspondence $F^{m_2} : \mathcal{F} \rightarrow 2^I$ where $F^{m_2}(f) \cap F^{m_2}\left(f'\right) = \emptyset$ for all f, f' in $\mathcal{F}$, m_2 in $\mathcal{M}_2$ and $\cup_{\mathcal{F}} F^{m_2}(f) = \mathcal{I}$ for all m_2 in $\mathcal{M}_2$. The correspondence F^{m_2} represents the assignment of consumers to ørms for the consumption of commodity m_2. Consumers may choose to purchase each commodity in $\mathcal{M}_2$ from only one ørm but we also allow consumers to purchase diœerent commodities from diœerent ørms. Therefore each function F^{m_2} may diœer for each commodity in $\mathcal{M}_2$. An allocation of consumers consists of the collection of functions $\left(F^{m_2}\right)_{\mathcal{M}_2}$. Let the allocation of consumers be S where $S = \left(\left(F^{m_2}(f)\right)_{\mathcal{F}}\right)_{\mathcal{M}_2}$. The proøle of consumers consuming from ørm f is $S^f = \left(F^{m_2}(f)\right)_{\mathcal{M}_2}$. The set of all such allocations is Z.

Each consumer i has a consumption set over the space of commodities relative to the allocation S, X_S^i where $X_S^i \subset \mathcal{R}^M$. The consumption set over the space of commodities and the allocations of consumers is $X^i \subset \mathcal{R}^M \times Z$. The preferences of consumer i are represented by a complete preordering $\succ^i$over X^i.

The consumption of private commodities by consumer i under the allocation S is x_S^i.

To keep the analysis simple consumers are only endowed with commodities in $\mathcal{M}_1$. The endowment of consumer i is w^i, where w^i is in X_S^i for all S. The aggregate endowment is w. Each consumer i has a shareholding θ^{if} in ørm f.

The following notational convention will sometimes be adopted: the price that each firm charges for commodities indexed by m_2 in $\mathcal{M}_2$ may differ so the commodity space will sometimes be expanded to $\mathcal{R}^{M_1+M_2F}$. In this case the consumption vector of consumer i relative to the allocation S is ax^i_S where

$$ax^i_S \;=\; ((x^{im_1}_{\mathcal{M}_1}), \underbrace{(0,\ldots,\underbrace{x^{iM_1+1}_S}_{f_{M_1+1}\ \texttt{th place}},\ldots,0)}_{F},\ldots,\underbrace{(0,\ldots,\underbrace{x^{iM_2}_S}_{f_{M_2}\ \texttt{th place}},\ldots,0)}_{F})$$

and where consumer i purchases any commodity m_2 in $\mathcal{M}_2$ from firm f_{m_2} in $\mathcal{F}$. From the consumption sets of consumer i, X^i_S and X^i we may generate in the expanded commodity space $\mathcal{R}^{M_1+M_2F}$ the consumption sets AX^i_S and AX^i in the obvious fashion.

2.2. Production

The set of production opportunities of firm f in the space of commodities relative to the profile of consumers of firm f is S^f where $Y^f_{S^f} \subset \mathcal{R}^M$. Nothing is lost by writing Y^f_S. The production set over the space of commodities and the allocation of consumers is $Y^f \subset \mathcal{R}^M \times Z$. The net output of firm f of commodities relative to the allocation S is y^f_S.

Aggregate production opportunities in the space of commodities relative to the allocation S are $Y_S \subset \mathcal{R}^M$. Aggregate production opportunities in the space of commodities and the allocation of consumers is $Y \subset \mathcal{R}^M \times Z$. The net output of firm f of commodities relative to the allocation S is y_S.

The marginal rate of transformation between commodities in $\mathcal{M}_1$ is independant of S.

The following notational convention will sometimes be adopted: in the expanded commodity space $\mathcal{R}^{M_1+M_2F}$ the net output vector of firm f is ay^f_S where

$$ay^f_S = ((y^{fm_1}_{\mathcal{M}_1}), \underbrace{(0,\ldots,\underbrace{y^{fM_1+1}_S}_{f\ \texttt{th place}},\ldots,0)}_{F},\ldots,\underbrace{(0,\ldots,\underbrace{y^{fM_2}_S}_{f\ \texttt{th place}},\ldots,0)}_{F}).$$

From the production sets Y_S^f, Y^f, Y_S and Y we may generate in the expanded commodity space $\mathcal{R}^{M_1+M_2F}$ the production sets AY_S^f, AY^f, AY_S and AY in the obvious fashion.

2.3. Prices and Complete Prices

The price space, Δ, consists of a price for each commodity in $\mathcal{M}_1$, p^{m_1}, a ørm speciøc price relative to the allocation of consumers S for each commodity, $p_S^{m_2f}$, a system of lump sum transfers between consumers relative to the allocation of consumers S, τ_S^i, and a system of lump sum transfers between ørms relative to the allocation of consumers S, τ_S^f. Let the price space be Δ, where

$$\Delta = \left\{ \left(p, \tau^C, \tau^P\right) \in \mathcal{R}^{M_1+M_2F} \times \mathcal{R}^I \times \mathcal{R}^F \mid \left(p, \tau^C, \tau^P\right) \neq 0 \right\}$$

Let the price of every commodity relative to the allocation of consumers S be p_S where $p_S = \left(\{p^{m_1}\}_{\mathcal{M}_1}, \left\{p_S^{m_2f}\right\}_{\mathcal{M}_2\mathcal{F}} \right)$. In addition let τ_S^C be the lump sum payments of all consumers where $\tau_S^C = (\tau_S^i)_{\mathcal{I}}$ and τ_S^P and the lump sum payments of all ørms where $\tau_S^P = \left(\tau_S^f\right)_{\mathcal{F}}$. Of course, $\sum_{\mathcal{I}} \tau_S^i - \sum_{\mathcal{F}} \tau_S^f = 0$ for every S.

Prices are complete if the price of all commodities in $\mathcal{M}_2$ and all lump sum payments may adjust as any consumer changes the ørms from whom he (she) purchases commodities. Prices are uniform if no commodity price or lump sum payments change as any consumer changes the ørms from whom he (she) purchases commodities and if the price each ørm charges for all commodities in $\mathcal{M}_2$ is the same. A uniform price system is denoted by p^* in Δ.

2.4. Feasibility

An allocation $\left((x_{S^*}^{*i})_{\mathcal{I}}, \left(y_{S^*}^{*f}\right)_{\mathcal{F}} \right)$ is a quasi-equilibrium relative to a complete price system $(p_S^*)_Z$ with lump sum transfers $\left(\tau_S^{*C}\right)_{\mathcal{Z}}$ and $\left(\tau_S^{*P}\right)_{\mathcal{Z}}$ if and only if

(1) for every f and for all y_S^f in Y_S^f, $p_{S^*}^* a y_{S^*}^{*f} + \tau_{S^*}^{*f} \geq p_S^* a y_S^f + \tau_S^{*f}$,
(2) for all i, if $x_S^i \succeq^i x_{S^*}^{*i}$ then $p_S^* a x_S^i + \tau_S^{*i} \geq p_{S^*}^* a x_{S^*}^{*i} + \tau_{S^*}^{*i}$,
(3) $y_{S^*}^* + w = x_{S^*}^*$.

A quasi-equilibrium relative to a uniform price system p^* is deøned as above. The allocation $\left((x_{S^*}^{*i})_{\mathcal{I}}, \left(y_{S^*}^{*f}\right)_{\mathcal{F}} \right)$ is an equilibrium relative to a complete price system $(p_S^*)_Z$ with lump sum transfers $\left(\tau_S^{*C}\right)_{\mathcal{Z}}$ and $\left(\tau_S^{*P}\right)_{\mathcal{Z}}$ if and only if (1), $(2')$ and (3) hold where $(2')$ is

$(2')$ for all i, if $x_S^i \succ^i x_{S^*}^{*i}$ then $p_S^* a x_S^i + \tau_S^{*i} > p_{S^*}^* a x_{S^*}^{*i} + \tau_{S^*}^{*i}$.

2.5. Pareto Optimality

Given the preferences of consumer i the better than, worse than and strictly better than sets relative to the allocation of consumers S are deøned as follows:

$$R_S^i\left(x_{S'}\right) = \left\{z_S \in X_S^i \mid z_S \succeq^i x_{S'}\right\}, L_S^i\left(x_{S'}\right) = \left\{z_S \in X_S^i \mid x_{S'} \succeq^i z_S\right\}$$

$$\text{and } P_S^i\left(x_{S'}\right) = \left\{z_S \in X_S^i \mid z_S \succ^i x_{S'}\right\}$$

An allocation $\left((x_{S^*}^{*i})_{\mathcal{I}}, \left(y_{S^*}^{*f}\right)_{\mathcal{F}}\right)$ is Pareto optimal if it is feasible and condition (3) holds.

(3) if there exists an allocation $\left((x_{S'}^{\prime i})_{\mathcal{I}}, \left(y_{S'}^{\prime f}\right)_{\mathcal{F}}\right)$ such that for all i, $x_{S'}^{\prime i} \succeq^i x_{S^*}^{*i}$ and for some i, $x_{S'}^{\prime i} \succ^i x_{S^*}^{*i}$ then $\left((x_{S'}^{\prime i})_{\mathcal{I}}, \left(y_{S'}^{\prime f}\right)_{\mathcal{F}}\right)$ cannot be feasible.

3. Example

In Example 3.1 supporting prices are found for a Pareto optimal allocation in a model with two consumers. It is shown that ımigrationȷ, from one ørm to another, will necessitate a change in the relative per unit prices faced by both consumers.

Example 3.1 There are two consumers indexed by i in $\mathcal{I} = \{1, 2\}$, two ørms indexed by f in $\mathcal{F} = \{1, 2\}$ and two commodities, leisure denoted by $l \in \mathcal{R}$ and ski-runs denoted by $r \in \mathcal{R}$. The consumption vector of consumer i relative to the allocation S is $x_S^i = (l_S^i, r_S^i)$.

Without loss of generality, consider the two allocations S_1 and S_2, respectively associated with both consumers purchasing from ørm 1 and consumer 1 purchasing from ørm 1 and consumer 2 purchasing from ørm 2. Let $Z = \{S_{1,} S_2\}$. Each consumer has a consumption set $X^i = \mathcal{R}_+^2 \times Z$.

The model is endowed with two units of leisure. The preference preordering of consumer 1 is represented by $U^1\left(l, r\right) = l + 2r$ and the preference preordering of consumer 2 is represented by $U^2\left(l, r\right) = l + 1/2r$ when consumers ski apart and $U^1\left(l, r\right) = 3/2\left(l + r\right)$ and $U^2\left(l, r\right) = 3/4\left(l + r\right)$ when consumers ski together.

The aggregate production sets deøned relative to the allocations S_1 and S_2, Y_{S_1} and Y_{S_2} respectively, are generated by $AY_{S_k}+w = \{ay \in \mathcal{R}^3 \mid ay = (2,(1,1))\}$ for k in $\{1,2\}$. The allocation $\left(x^{*1}_{S_2}, x^{*2}_{S_2}\right) = ((1,(1,0)),(1,(0,1)))$ is Pareto optimal. At the allocation $\left(x^{*1}_{S_2}, x^{*2}_{S_2}\right)$ consumers enjoy utility of $U^1\left(x^{*1}_{S_2}\right) = 3$ and $U^2\left(x^{*2}_{S_2}\right) = 3/2$. Therefore the set of commodity allocations that would leave both consumers at least as well oœ as at $\left(x^{*1}_{S_2}, x^{*2}_{S_2}\right)$ is $P_{S_1} \cup P_{S_2}$ where

$$P_{S_1} = \Big\{ax_{S_1} \in AX_{S_1} \mid ax_{S_1} = (l, r_{S_1}, 0), l = l^1 + l^2, r_{S_1} = r^1_{S_1} + r^2_{S_1}, .$$

$$l^1 + 2r^1_{S_1} \geq 3 \text{ and } l^2 + 1/2r^2_{S_1} \geq 3/2\Big\},$$

$$P_{S_2} = \Big\{ax_{S_2} \in AX_{S_2} \mid ax_{S_2} = \left(l, r^1_{S_2}, r^1_{S_2}\right), l = l^1 + l^2,$$

$$3/2\left(l^1 + r^1_{S_2}\right) \geq 3 \text{ and } 3/4\left(l^2 + r^2_{S_2}\right) \geq 3/2\Big\}.$$

Pick leisure to be numeraire. Let the set of admissable seperating prices under each partition, S_k, k in $(1,2)$, be $\Delta^C_{S_k}$, where

$$\Delta^C_{S_k} = \left\{\left(p_{S_k}, \tau^C_{S_k}, \tau^P_{S_k}\right) \in \mathcal{R}^7 \mid p_{S_k} = \left(1, \left(p^1_{S_k}, p^2_{S_k}\right)\right) \neq 0\right\}.$$

By inspection

$$\Delta^C_{S_1} = \left\{\left(p_{S_1}, \tau^C_{S_1}, \tau^P_{S_1}\right) \in \mathcal{R}^7 \mid p_{S_1} = (1,(1,1)), \tau^C_{S_1} = (-1,2) \text{ and } \tau^P_{S_1} = (0,1/2)\right\}$$

and

$$\Delta^C_{S_2} = \left\{\left(p_{S_2}, \tau^C_{S_2}, \tau^P_{S_2}\right) \in \mathcal{R}^7 \mid p_{S_1} = (1,(2,1/2)), \tau^C_{S_1} = (0,0) \text{ and } \tau^P_{S_1} = (0,0)\right\}.$$

4. Welfare Theorems

That any equilibrium relative to a system of complete prices and lump sum transfers is Pareto optimal is demonstrated for the class of economies that satisfy the following assumptions in Theorem 1.

a.1 for every i and for every S, X_S^i is convex,

a.2 for every i and for every S, if x_S^{1i} and x_S^{2i} are two points of X_S^i and if t is a real number in $(0,1)$ then $x_S^{1i} \succ^i x_S^{2i}$ implies $tx_S^{1i} + (1-t)\, x_S^{2i} \succ^i x_S^{1i}$.

Theorem 1. Under assumptions a.1 and a.2 if $\left((x_{S*}^{*i})_{\mathcal{I}}, \left(y_{S*}^{*f}\right)_{\mathcal{F}}\right)$ is an equilibrium relative to a system of complete prices $(p_S^*)_Z$ and lump sum transfers $\left(\tau_S^{C*}\right)_Z$ and $\left(\tau_S^{F*}\right)_Z$, such that $\sum_I \tau_S^{*i} - \sum_F \tau_S^{*f} = 0$, for all S, then $\left((x_{S*}^{*i})_{\mathcal{I}}, \left(y_{S*}^{*f}\right)_{\mathcal{F}}\right)$ is Pareto optimal.

That any Pareto optimal allocation $\left((x_{S*}^{*i})_{\mathcal{I}}, \left(y_{S*}^{*f}\right)_{\mathcal{F}}\right)$ may be implemented as an equilibrium relative to a complete price system with lump sum transfers is demonstrated for the class fo economies that satisfy assumptions a.1, a.2 and the following assumptions, in Theorem 2.

a.3 for every i and for every S, $R_S^i(x_{S*}^{*i})$ and $L_S^i(x_{S*}^{*i})$ are closed in X_S^i,

a.4 for every i, $w_S^i \in X_S^i$ and $0 \in Y_S$, for every S,

a.5 for every S, Y_S is convex.

Theorem 2. Under assumptions a.1 through a.5, if $\left((x_{S*}^{*i})_{\mathcal{I}}, \left(y_{S*}^{*f}\right)_{\mathcal{F}}\right)$ is Pareto optimal there exists a price vector $(p_S^*)_Z$ and lump sum transfers $\left(\tau_S^{C*}\right)_Z$ and $\left(\tau_S^{F*}\right)_Z$, where $\sum_I \tau_S^{*i} - \sum_F \tau_S^{*f} = 0$, for all S, $\tau_{S*}^{*i} = 0$ for all i and $\tau_{S*}^{*f} = 0$ for all f, such that $\left((x_{S*}^{*i})_{\mathcal{I}}, \left(y_{S*}^{*f}\right)_{\mathcal{F}}, (p_S^*)_Z, \left(\tau_S^{C*}\right)_Z, \left(\tau_S^{F*}\right)_Z\right)$ is a quasi-equilibrium relative to a complete price system with lump sum transfers.

Let $A(Y)$ be the asymptotic cone of Y, that is $A(Y) = \{\alpha y \mid y \in Y \text{for all} \alpha \geq 0\}$.

a.6 for every S, $R_S(x_{S*}^*) \cap \{A(Y_S) + w\} = \emptyset$.

Theorem 3. Under assumptions a.1 through a.6, if $\left((x_{S*}^{*i})_{\mathcal{I}}, \left(y_{S*}^{*f}\right)_{\mathcal{F}}\right)$ is Pareto optimal there exists a price vector $(p_S^*)_Z$ and lump sum transfers $\left(\tau_S^{C*}\right)_Z$ and $\left(\tau_S^{F*}\right)_Z$, where $\sum_I \tau_S^{*i} = 0, \sum_F \tau_S^{*f} = 0$, for all S, $\tau_{S*}^{*i} = 0$ for all i and $\tau_{S*}^{*f} = 0$ for all f and such that $\left((x_{S*}^{*i})_{\mathcal{I}}, \left(y_{S*}^{*f}\right)_{\mathcal{F}}, (p_S^*)_Z, \left(\tau_S^{C*}\right)_Z, \left(\tau_S^{F*}\right)_Z\right)$ is a quasi-equilibrium relative to a complete price system with lump sum transfers.

If, in addition, assumption a.7 holds we can constrain prices to a system of uniform prices that is independant of the allocation of consumers.

a.7 for every S, $Y_S \subseteq Y_{S*}$ and $R_S(x^*_{S*}) \subseteq R_{S*}(x^*_{S*})$.

Theorem 4. Under assumptions a.1 through a.5 and a.7 , if $\left((x^{*i}_{S*})_{\mathcal{I}}, \left(y^{*f}_{S*}\right)_{\mathcal{F}}\right)$ is Pareto optimal there exists a price vector $(p^*_S)_Z$ and lump sum transfers $\left(\tau^{C*}_S\right)_Z$ and $\left(\tau^{F*}_S\right)_Z$, where $\sum_I \tau^{*i}_S = 0, \sum_F \tau^{*f}_S = 0$, for all S, $\tau^{*i}_{S*} = 0$ for all i and $\tau^{*f}_{S*} = 0$ for all f and such that $\left((x^{*i}_{S*})_{\mathcal{I}}, \left(y^{*f}_{S*}\right)_{\mathcal{F}}, (p^*_S)_Z, \left(\tau^{C*}_S\right)_Z, \left(\tau^{P*}_S\right)_Z\right)$ is a quasi-equilibrium relative to a complete price system relative to a uniform price system.

Theorem 5. If $\left((x^{*i}_{S*})_{\mathcal{I}}, \left(y^{*f}_{S*}\right)_{\mathcal{F}}\right)$ is Pareto optimal then for every i where there exists $\widetilde{x}^i_S$ in X^i_S such that $p^*_S\widetilde{x}^i_S + \tau^{*i}_S < p^*_{S*}x^{*i}_{S*} + \tau^{*i}_{S*}$ for every S, where $(p^*_S)_Z, (\tau^{*i}_S)_Z$ is established in Theorem 2, 3 or 4, then we have that if $x^i_S \succ^i x^{*i}_{S*}$ then $p^*_S\widetilde{x}^i_S + \tau^{*i}_S > p^*_{S*}x^{*i}_{S*} + \tau^{*i}_{S*}$.

5. Existence of Equilibrium

An allocation $\left((x^{*i}_{S*})_{\mathcal{I}}, \left(y^{*f}_{S*}\right)_{\mathcal{F}}\right)$ is an equilibrium at complete prices $(p^*_S)_Z$ and lump sum transfers $\left(\tau^{C*}_S\right)_Z$ and $\left(\tau^{P*}_S\right)_Z$,where $\sum_{\mathcal{I}} \tau^{*i}_S - \sum_{\mathcal{F}} \tau^{*f}_S = 0$, for all S, if and only if

(1) for every f and for all y^f_S in Y^f_S, $p^*_{S*}ay^{*f}_{S*} + \tau^{*f}_{S*} \geq p^*_S a y^f_S + \tau^{*f}_S$,

(2) for all i, $p^*_{S*}ax^{*i}_{S*} + \tau^{*i}_{S*} \leq p^*_{S*}w^i + \sum_{\mathcal{F}} \theta^{if}\left(p^*_{S*}ay^{*f}_{S*} + \tau^{*f}_{S*}\right)$ and for all x^i_S such that $x^i_S \succ^i x^{*i}_{S*}$, $p^*_S a x^i_S + \tau^{*i}_S > p^*_{S*}ax^{*i}_{S*} + \tau^{*i}_{S*}$,

(3) $y^*_{S*} + w = x^*_{S*}$.

Generally the proof of existence for the model in Section 2 is diŒcult. The reasons are presented in Manning (1993a) and (1993b) in the context of a class of models with local public goods. They are repeated here, in part, for completeness. Commodities are indexed by the allocation with which they are associated. In this commodity space the projection of the net production set, $\Pi_Z Y_S$, into the private good subspace is star convex relative to the endowment point. Production may never occur under more than one allocation. This generates discontinuities in the demand and supply correspondences as prices change. As prices change consumers can change the ørm they purchase from. As a consumer ımigratesȷ his (her) demand correspondence for the output of the former ørm takes on the value zero. Therefore the aggregate demand and supply correspondences under the former allocation take on the value zero.

However a technique for avoiding such discontinuities suggests itself. To prove the existence of an Equilibrium at complete prices, given the Second Welfare Theorem holds, it is suŒcient to prove that for some Pareto optimal allocation $(x_{S*}^{*i})_{\mathcal{I}}$,

$$\text{for every } i, p_{S*}^{*} a x_{S*}^{*i} + \tau_{S*}^{*i} \leq p_{S*}^{*} w^{i} + \sum_{\mathcal{F}} \theta^{if} \left(p_{S*}^{*} a y_{S*}^{*f} + \tau_{S*}^{*f} \right).$$

The complete price system $\left((p_S^*)_Z, (\tau_S^{*i})_{\mathcal{I}Z}, \left(\tau_S^{*f}\right)_{\mathcal{F}Z} \right)$ is a selection from the set of complete prices that support the Pareto optimal allocation $(x_{S*}^{*i})_{\mathcal{I}}$ is the sense deøned in Section 2.

If all Pareto optimal allocations are associated with one allocation, the discontinuities associated with the search for a Pareto optimal allocation that satisfy (1) can be avoided.

a.8 all Pareto optimal allocations are associated with one partition, S^* say,
a.9 for every i, X_{S*}^{i} is bounded below in $\leq$,
a.10 for every i, X_{S*}^{i} is closed,
a.11 for every i and for every x_{S*}^{*i} in X_{S*}^{i}, there is an x_{S*}^{i} in X_{S*}^{i} preferred to x_{S*}^{*i},
a.12 the relative interiors of $Y_{S*} + w$ and X_{S*} have non-empty intersection,
a.13 Y_{S*} is closed,
a.14 $Y_{S*} \cap \mathcal{R}_{+}^{M} = \{0\}$.

Theorem 6. Under assumptions a.1 through a.3, a.6 through a.12 and the cheaper point assumption of Theorem 5, there exists an allocation $\left((x_{S*}^{*i})_{\mathcal{I}}, \left(y_{S*}^{*f}\right)_{\mathcal{F}} \right)$ that is an equilibrium at complete prices $(p_S^*)_Z$ and lump sum transfers $\left(\tau_S^{C*}\right)_Z$ and $\left(\tau_S^{P*}\right)_Z$,where $\sum_{\mathcal{I}} \tau_S^{*i} - \sum_{\mathcal{F}} \tau_S^{*f} = 0$, for all S, $\tau_S^{*i} = 0$ for all i and $\tau_S^{*f} = 0$ for all f.

Theorem 7. Under assumptions a.1 through a.4, a.6 through a.12 and the cheaper point assumption of Theorem 5, there exists an allocation $\left((x_{S*}^{*i})_{\mathcal{I}}, \left(y_{S*}^{*f}\right)_{\mathcal{F}} \right)$ that is an equilibrium at complete prices $(p_S^*)_Z$ and lump sum transfers $\left(\tau_S^{C*}\right)_Z$ and $\left(\tau_S^{P*}\right)_Z$,where $\sum_{\mathcal{I}} \tau_S^{*i} = 0$, $\sum_{\mathcal{F}} \tau_S^{*f} = 0$, for all S, $\tau_S^{*i} = 0$ for all i and $\tau_S^{*f} = 0$ for all f.

Theorem 8. Under assumptions a.1 through a.4, a.6 through a.13 and the cheaper point assumption of Theorem 5, there exists an allocation $\left((x^{*i}_{S^*})_{\mathcal{I}}, \left(y^{*f}_{S^*}\right)_{\mathcal{F}}\right)$ that is an equilibrium at a uniform price p^*.

6. The Core

A coalition is denoted by $\mathcal{C} \subseteq \mathcal{I}$. For each commodity there is a function $F^{m_2}_{\mathcal{C}} : \mathcal{F} \to 2^{\mathcal{C}}$ where $F^{m_2}_{\mathcal{C}}(f) \cap F^{m_2}_{\mathcal{C}}(f') = \emptyset$ for all f, f' in $\mathcal{F}$, m_2 in $\mathcal{M}_2$ and $\cup_F F^{m_2}_{\mathcal{C}}(f) = \mathcal{C}$ for all m_2 in $\mathcal{M}_2$. $F^{m_2}_{\mathcal{C}}(f)$ represents the members of coalition $\mathcal{C}$ that purchase commodity m_2 form ørm f. An allocation of consumers in coalition $\mathcal{C}$ consists of the collection of functions $(F^{m_2}_{\mathcal{C}})_{\mathcal{M}_2}$. The proøle of consumers in coalition $\mathcal{C}$ consuming from ørm f is $S^f_{\mathcal{C}} = (F^{m_2}_{\mathcal{C}}(f))_{\mathcal{M}_2}$. Denote the allocation of consumers in coalition $\mathcal{C}$ by $S_{\mathcal{C}}$ where $S^f_{\mathcal{C}} = ((F^{m_2}_{\mathcal{C}}(f))_F)_{M_2}$. The set of all such allocations is $Z_{\mathcal{C}}$.

Each consumer i has a consumption set over the space of commodities relative to the allocation $S_{\mathcal{C}}$, $X^i_{S_{\mathcal{C}}}$ where $X^i_{S_{\mathcal{C}}} \subset \mathcal{R}^M$. The consumption set over the space of commodities and the allocations of consumers is $X^i_{\mathcal{C}} \subset \mathcal{R}^M \times Z_{\mathcal{C}}$. Preferences for each consumer i are represented by a complete preordering $\succ^i$over $X^i_{\mathcal{C}}$ for all $\mathcal{C}$.

The consumption vector of consumer i as a member of the coalition $\mathcal{C}$ is denoted $x^i_{S_{\mathcal{C}}}$ in $X^i_{S_{\mathcal{C}}}$.

The production opportunities of ørm f in a coalition $\mathcal{C}$ relative to the proøle $S^f_{\mathcal{C}}$ are $Y^f_{S^f_{\mathcal{C}}}$ where $Y^f_{S^f_{\mathcal{C}}} \subset R^M$. Nothing is lost by writing $Y^f_{S_{\mathcal{C}}}$. The production set over the space of commodities and the allocation of consumers is $Y^f_{\mathcal{C}} \subset R^M \times Z_{\mathcal{C}}$. An element of $Y^f_{S_{\mathcal{C}}}$ is denoted by $y^f_{S_{\mathcal{C}}}$.

An allocation $\left(\left(x^{*i}_{S^*_{\mathcal{C}}}\right)_{\mathcal{C}}, \left(y^{*f}_{S^*_{\mathcal{C}}}\right)_{\mathcal{F}}\right)$ is $\mathcal{C}$-feasible if

(1) for all i in $\mathcal{C}$, $x^{*i}_{S^*_{\mathcal{C}}} \in X^i_{\mathcal{C}}$ and for all f, $y^{*f}_{S^*_{\mathcal{C}}} \in Y^f_{S_{\mathcal{C}}}$.

(2) $y^*_{S^*_{\mathcal{C}}} + \sum_{\mathcal{C}} w^i = x^*_{S^*_{\mathcal{C}}}$.

An allocation $\left(\left(x^{*i}_{S^*_{\mathcal{C}}}\right)_{\mathcal{C}}, \left(y^{*f}_{S^*_{\mathcal{C}}}\right)_{\mathcal{F}}\right)$ is blocked by a coalition $\mathcal{C} \neq \emptyset$ if there exists a $\mathcal{C}$-feasible allocation $\left(\left(x'^i_{S'_{\mathcal{C}}}\right)_{\mathcal{C}}, \left(y'^f_{S'_{\mathcal{C}}}\right)_{\mathcal{F}}\right)$ such that

(3) $x'^i_{S'_{\mathcal{C}}} \succeq^i x^{*i}_{S^*_{\mathcal{C}}}$ for all i in C and $x'^i_{S'_{\mathcal{C}}} \succ^i x^{*i}_{S^*_{\mathcal{C}}}$ for some i in $\mathcal{C}$.

An allocation is in the core if it cannot be blocked.

a.15 for every $\mathcal{C}$ and i, $X^i_{S_\mathcal{C}}$ is convex for all $S_\mathcal{C}$,

a.16 for every $\mathcal{C}$ and i and every $x^i_{S_\mathcal{C}}$ in $X^i_{S_\mathcal{C}}$ there is a commodity bundle $x'^i_{S'_\mathcal{C}}$ such that $x'^i_{S'_\mathcal{C}} \succ^i x^i_{S_\mathcal{C}}$, for all $S_\mathcal{C}$.

a.17 for any C and i, let $x'^i_{S_\mathcal{C}}$ and $x^i_{S_\mathcal{C}}$ be arbitrary diœerent commodity bundles in $X^i_{S_\mathcal{C}}$ with $x'^i_{S_\mathcal{C}} \succeq^i x^i_{S_\mathcal{C}}$, and let $\alpha \in (0,1)$. We assume that $\alpha x'^i_{S_\mathcal{C}} + (1-\alpha) x^i_{S_\mathcal{C}} \succ^i x^i_{S_\mathcal{C}}$, for all $S_\mathcal{C}$.

a.18 for every $\mathcal{C}$ and f, Y^f_{S*} and $Y^f_{S_\mathcal{C}}$, for all $S_\mathcal{C}$, are convex cones with vertex at the origin.

a.19 for every $\mathcal{C}$ and f, $Y^f_{S_\mathcal{C}} \subseteq Y^f_{S*}$ for all $S_\mathcal{C}$.

Assumption 19 says that any blocking coalition $\mathcal{C}$ cannot produce some vector of private goods that the grand coalition cannot.

Theorem 9. Under assumptions a.15 through a.19 $\left(\left(x^{*i}_{S*}\right)_\mathcal{I}, \left(y^{*f}_{S*}\right)_\mathcal{F}, (p^*_S)_Z, \left(\tau^{C*}_S\right)_Z, \left(\tau^{P*}_S\right)_Z\right)$ is an equilibrium at complete prices $(p^*_S)_Z$ and lump sum transfers $\left(\tau^{C*}_S\right)_Z$ and $\left(\tau^{P*}_S\right)_Z$, where $\tau^{*i}_{S*} = 0$ for all i and $\tau^{*f}_{S*} = 0$ for all f, then $\left(\left(x^{*i}_{S*}\right)_\mathcal{I}, \left(y^{*f}_{S*}\right)_\mathcal{F}\right)$ is in the core.

From Theorem 9 follows the First Welfare Theorem for the class of models that exhibit constant returns to scale.

7. Discussion

Complete prices are manipulable when the number of agents is small. Avoiding manipulation requires assuming that the Social Planner know the preferences of consumers and the production opportunities open to ørms. If the Social Planner is not perfectly informed about consumer preferences or the production opportunities of ørms then it is an open question what mechanism would allow the Social Planner to avoid the manipulation.

The results in this paper indicate important limitations to standard techniques for evaluating the beneøts (costs) of the class of commodities with the nonconvexities described. Since complete prices are required for the evaluation of many private projects and complete prices often do not, in reality, exist the correct complete (ıshadowȷ) prices must be constructed. We have shown that in constructing these complete prices for some private good supplied by some ørm information about the allocation of consumers among ørms and the output of other ørms may need to be incorporated. Theorem 3 and Theorem 4 indicate that, under restrictive conditions, these informational requirements can be relaxed.

8. Appendix

Let $R_S\left((x^{*i}_{S^*})_{\mathcal{I}}\right) = \sum_{\mathcal{I}} R^i_S\left(x^{*i}_{S^*}\right)$. Let

$$AY^f_S = \left\{\left(\left(y^{fm_1}\right)_{\mathcal{M}_1}, \left(0, \ldots, y^{fM_1+1}_S, \ldots, 0\right), \ldots, \left(0, \ldots, y^{fM_2}_S, \ldots, 0\right)\right) \in R^{M_1+M_2F} \mid \left(\left(y^{fm_1}_S\right)_{\mathcal{M}_1}, \left(y^{fm_2}_S\right)_{\mathcal{M}_2}\right) \in Y^f_S\right\}.$$

The aggregate production set $AY_S = \sum_{\mathcal{F}} AY^f_S$. Let:

$$AR^i_S\left(x^{*i}_{S^*}\right) = \left\{\left(\left(x^{fm_1}\right)_{\mathcal{M}_1}, \left(0, \ldots, x^{iM_1+1}_S, \ldots, 0\right), \ldots, \left(0, \ldots, x^{iM_2}_S, \ldots, 0\right)\right) \in R^{M_1+M_2F} \mid x^i_S \in X^i_S \text{ and } x^i_S \succeq^i x^{*i}_{S^*} \text{ for all } S\right\}.$$

Let:

$$AP^i_S\left(x^{*i}_{S^*}\right) = \left\{\left(\left(x^{im_1}\right)_{\mathcal{M}_1}, \left(0, \ldots, x^{iM_1+1}_S, \ldots, 0\right), \ldots, \left(0, \ldots, x^{iM_2}_S, \ldots, 0\right)\right) \in R^{M_1+M_2F} \mid x^i_S \in X^i_S \text{ and } x^i_S \succ^i x^{*i}_{S^*} \text{ for all } S\right\}.$$

Often $AR^i_S\left(x^{*i}_{S^*}\right)$ will be written AR^i_S and $AP^i_S\left(x^{*i}_{S^*}\right)$ will be written AP^i_S for short. Let $G^i_S = AP^i_S\left(x^{*i}_{S^*}\right) + \sum_{\mathcal{I}\setminus i} AR^i_S\left(x^{*i}_{S^*}\right) - \sum_{\mathcal{F}} AY^f_S$.

Proof of Theorem 1: Let $\left(\left(x^{*i}_{S^*}\right)_{\mathcal{I}}, \left(y^{*f}_{S^*}\right)_{\mathcal{F}}\right)$ be an equilibrium relative to a system of complete prices $\left(p^*_S\right)_Z$ and lump sum transfers $\left(\tau^{C*}_S\right)_Z$ and $\left(\tau^{F*}_S\right)_Z$. Suppose that $\left(\left(x^{*i}_{S^*}\right)_{\mathcal{I}}, \left(y^{*f}_{S^*}\right)_{\mathcal{F}}\right)$ is not Pareto optimal. Then there exists a feasible allocation $\left(\left(x'^i_{S'}\right)_{\mathcal{I}}, \left(y'^f_{S'}\right)_{\mathcal{F}}\right)$ such that $x'^i_{S'} \succ^i x^{*i}_{S^*}$for some i and $x'^i_{S'} \succeq^i x^{*i}_{S^*}$for all i. Therefore $\sum_{\mathcal{I}}\left(p_{S'} a x'^i_{S'} + \tau^{*i}_{S'}\right) > \sum_{\mathcal{I}}\left(p^*_{S^*} a x^{*i}_{S^*} + \tau^{*i}_{S^*}\right)$. It follows immediately that $\sum_{\mathcal{F}}\left(p^*_{S'} a y^f_{S'} + \tau^{*f}_{S'}\right) > \sum_{\mathcal{F}}\left(p^*_{S^*} a y^{*f}_{S^*} + \tau^{*f}_{S^*}\right)$ contradicting proøt maximisation.‖

Proof of Theorem 2: Since the state $\left(\left(x^{*i}_{S^*}\right)_{\mathcal{I}}, \left(y^{*f}_{S^*}\right)_{\mathcal{F}}\right)$ is Pareto optimal, w does not belong to G^i_S, for every S. If follows from 1,2 and 3 that the sets AP^i_S and AR^i_S are convex for every S. Hence G^i_S is convex as the sum of convex sets for every S. Thus, by Minkowski's theorem, there is a hyperplane H_S through w_S, bounding for G^i_S, for every S i.e. there is a p_S in $\mathcal{R}^{M_1+M_2F}$ diœerent from 0 such that $p_S b_S \geq w$ for every b_S in G^i_S and every S.

The set G_S is contained in the adherence of G_S^i and hence in the closed half-space above the hyperplane H_S, for every S. Since the point w belongs to G_{S^*} it minimises $p_{S^*}b_{S^*}$ on G_{S^*}. It follows from $w = \sum_I ax_{S^*}^{*i} - \sum_F ay_{S^*}^{*f}$ that $ax_{S^*}^{*i}$ that minimises $p_{S^*}b_{S^*}$ on $AR_{S^*}^i$ for every i and $ay_{S^*}^{*f}$ maximises $p_{S^*}b_{S^*}$ on $AY_{S^*}^f$ for every f.

We may write $p_{S^*}ax_{S^*}^* = p_{S^*}ay_{S^*}^* + p_{S^*}w$ and $p_Sax_S \geq p_Say_S + p_Sw$, for every $S \neq S^*$. There exists an r_S such that $p_Sax_S \geq r_S \geq p_Say_S + p_Sw$ for every S.

To show that $p_Sax_S^i + \tau_S^i \geq p_{S^*}ax_{S^*}^{*i} + \tau_{S^*}^i$ and $p_Say_S^f + \tau_S^f \leq p_{S^*}ay_{S^*}^{*f} + \tau_{S^*}'^f$ note that the net payments $(\tau_S^i)_{\mathcal{I}}$ and $\left(\tau_S^f\right)_{\mathcal{F}}$ may be of any sign. Therefore condition (1) and (2) of an equilibrium relative to a complete price system with lump sum transfers is satisøed.

Condition (3) of an equilibrium relative to a complete price system with lump sum transfers is immediate by the deønition of Pareto optimality. $\|$

Proof of Theorem 3: Since $R_S \cap \{A(Y_S) + w\} = \emptyset$ for every S, it is possible to pick prices such that total proøt is zero under each allocation. In this case we can let $r_S = p_Sw$ for every S and the Theorem follows. $\|$

Proof of Theorem 4: If assumption 6 holds there exists a common hyperplane through w, bounding for G_S^i, for every S. $\|$

Proof of Theorem 5: To prove the theorem it is suŒcient to show that if x_S^{2i} in X_S^i satisøes $p_S^{*i}ax_S^{2i} + \tau_S^{*i} = p_{S^*}^{*i}ax_{S^*}^{*i} + \tau_{S^*}^{*i}$ then $x_{S^*}^{1i} \succeq^i x_S^{2i}$. Consider any point x_S^i of the segment $[x_S^{1i}, x_S^{2i})$. Clearly, $p_S^{*i}ax_S^i + \tau_S^{*i} < p_{S^*}^{*i}ax_{S^*}^{*i} + \tau_{S^*}^{*i}$, hence, by (b) of Theorem 2, $x_{S^*}^{*i} \succ^i x_S^i$. Therefore x_S^{2i} is adherent to the set $\{x_S^i \in X_S^i \mid x_{S^*}^{*i} \succeq^i x_S^i\}$. As the latter is closed by continuity it owns x_S^{2i}.$\|$

Proof of Theorem 6: Under 1 through 4 the Pareto optimal allocation $((ax_{S^*}^{*i})_{\mathcal{I}}$, $\left(ay_{S^*}^{*f}\right)_{\mathcal{F}})$ has a supporting price. By assumption the value of each consumers endowment of the private good is invariant to the allocation with which his (her) residence is associated.

By 7 any Pareto optimal allocation is associated with a unique allocation S^*. Therefore, to prove the existence of an equilibrium it is suŒcient to prove the existence of an equilibrium under the allocation S^*.

Existence under the allocation S^* follows as 1 through 4, 6 and 8 through 13 imply the assumptions used to prove existence by Debreu (1962). 8 implies a.1, 1 and 9 imply a.2, 10 implies b.1, 3 implies b.2, 2 implies b.3, 11 implies c.1, 4 and 12 imply d.1 and 13 implies d.2. $\|$

Proof of Theorem 7: Immediate. $\|$

Proof of Theorem 8: Immediate. ‖

Proof of Theorem 9: Suppose that $\left((x^{*i}_{S*})_{\mathcal{I}}, \left(y^{*f}_{S*}\right)_{\mathcal{F}}\right)$ can be blocked by a coalition $\mathcal{C}$ with an allocation $\left(\left(x^i_{S_{\mathcal{C}}}\right)_{\mathcal{I}}, \left(y^f_{S_{\mathcal{C}}}\right)_{\mathcal{F}}\right)$. By 18, $p^*_{S*} a y^{*f}_{S*} \leq 0$ for every f in $\mathcal{F}$. Notice that $x^i_{S_{\mathcal{C}}} \succeq^i x^{*i}_{S*}$ implies $p^*_{S*} a x^i_{S_{\mathcal{C}}} \geq p^*_{S*} w^i$ for all i in $\mathcal{C}$, by 15, 16 and 17 and that $x^i_{S_{\mathcal{C}}} \succ^i x^{*i}_{S*}$ implies $p^*_{S*} a x^i_{S_{\mathcal{C}}} > p^*_{S*} w^i$ for some i in $\mathcal{C}$. That $\left(\left(x^i_{S_{\mathcal{C}}}\right)_{\mathcal{I}}, \left(y^f_{S_{\mathcal{C}}}\right)_{\mathcal{F}}\right)$ is $\mathcal{C}$-feasible implies $\sum_{\mathcal{C}} \left(x^i_{S_{\mathcal{C}}} - w^i\right) = \sum_{\mathcal{F}} y^f_{S_{\mathcal{C}}}$ where $y^f_{S_{\mathcal{C}}} \in Y^f_{S_{\mathcal{C}}}$ for all f in $\mathcal{F}$.

Since $p^*_{S*} a x^i_{S_{\mathcal{C}}} \geq p^*_{S*} w^i$ for all i with strict inequality for one i then $\sum_{\mathcal{C}} p^*_{S*} a x^i_{S_{\mathcal{C}}} \geq \sum_{\mathcal{C}} p^*_{S*} w^i$, for some i in $\mathcal{C}$. That $\left(\left(x^i_{S_{\mathcal{C}}}\right)_{\mathcal{I}}, \left(y^f_{S_{\mathcal{C}}}\right)_{\mathcal{F}}\right)$ is $\mathcal{C}$-feasible implies $\sum_{\mathcal{C}} \left(x^i_{S_{\mathcal{C}}} - w^i\right)$ $= \sum_{\mathcal{F}} y^f_{S_{\mathcal{C}}}$ where $y^f_{S_{\mathcal{C}}} \in Y^f_{S_{\mathcal{C}}}$ for all f in $\mathcal{F}$.

Since $p^*_{S*} a x^i_{S_{\mathcal{C}}} \geq p^*_{S*} w^i$ for all i with strict inequality for one i then $\sum_{\mathcal{C}} p^*_{S*} a x^i_{S_{\mathcal{C}}} \geq \sum_{\mathcal{C}} p^*_{S*} w^i$, or $p^*_{S*} a y_{S_{\mathcal{C}}} > 0$, a contradiction by 18 and 19. ‖

9. References

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